

# Quantifiable resilience analytics of power grids by simulating power flows

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New challenges are expected to emerge for the operation of power grids and associated power flows from the growing use of renewable energies and the increasing demand of electricity due to the electrification of mobility. Current stability considerations of power grid under normal operating conditions are presumably insufficient to handle contingencies. For this, the concept of resilience analysis offers advantages to study the operation under the influence of disturbances. However, there is still a lack of suitable approaches. This includes a lack of metrics for making quantitative statements about the grid resilience. This paper develops a qualitative concept of power grid resilience and complements it by a quantitative analysis method, which uses the modeling and simulation of power flows. This concept is illustrated by a case study. So, this paper makes a contribution to provide a suitable approach and a quantitative metric for power grid resilience.

*Keywords:* Resilience analytics, resilience metric, power grids, optimal power flow.

## 1. Introduction

The operation and design of power grids is facing new challenges. One of the challenges is to become prepared for extreme weather events, which are expected to become more likely. Also the increase in renewable energy will lead itself to multiple challenges. The obvious challenge there is to match the power production course with the power consumption course. But also, a decentralized energy production and storage will decrease the load on the grid, which makes it more difficult for the grid provider to keep the grid stable. The transportation of heavy loads will become more important as the industrial users will not be close to the energy production. Additionally, the possible electrification of mobility would lead to a huge increase of the total power demand, which would put the power grid under enormous stress.

To cope with these challenges, new concepts are needed. Resilience is a promising new concept, that recently got a lot of interest. In the most general sense, it is the behavior to react to deviations. Resilience does not have a precise broadly accepted definition yet or a strong standing in industry. This is in contrast to risk, reliability, or vulnerability. Active research in the field of resilience is wide-ranging and complex. In order to get a small glimpse of the current research, some works are presented in the following.

Bruneau et al. (2003) indicate robustness, re-

dundancy, resourcefulness, and rapidity as the four components of resilience. Furthermore, the authors indicate technical, organizational, social, and economic as four aspects of resilience, which are supposed to be measured and evaluated individually. The supposed three strategies to enhance resilience is to reduce failure probabilities, consequences from failures, and time to recover. This paper has additionally one of the first influential mentioning of the resilience triangle, that indicates the strength of a deviation and the time to recover.

Roeger et al. (2014) present a matrix-based approach to determine an energy resilience metric. This energy metric is not specific for power grids, but concerns hospitals, regional utility manager, military installations, and all that have an increased interest in energy resilience. The matrix has four columns and four rows. The columns of the matrix consists of four stages of resilience: plan, absorb, recover, and adapt. The rows of the matrix consists of four domains: physical, information, cognitive, and social. This metric helps decision makers to identify deficits in the energy resilience.

Panteli et al. (2017) introduce a framework to quantify resilience. The key concept is the assumption that the resilience is reflected in a trapezoidal curve of the resilience level. To quantify the resilience, four values are taken from the corners

of the resilience trapezoidal. These values indicate the resilience drop, the duration of the drop, the extensiveness of the post-disturbance phase, and the duration of the recovery. This framework is applied to a 29-bus system which is exposed to a wind storm. Two strategies to increase resilience are compared, on the one hand lines are strengthened to reduce the impact of the winds, and on the other hand the control is made smarter to decrease reaction times.

A recent overview over resilience of power grids can be found in Huang et al. (2017). Multiple definitions and measures are presented, which help to enhance resilience. The resilience enhancement is differentiated between resilience planning, resilience response and resilience restoration.

The presented paper is divided into 5 sections. In Section 2, the basic terms are introduced and defined. In Section 3, a method to quantify resilience is introduced and explained. In Section 4, a small case study is done to illustrate the developed method. In Section 5, the results are summarized and an outlook for future work is given.

## **2. Terms**

The terms risk, resilience, and reliability have a lot in common in their terminology and application, but there are also fundamental differences. In their semantics, three aspects can be roughly distinguished: The terms can be meant descriptively, they can specify a goal (as a desired system property), or they can be a measurement.

If one follows the established definitions of risk (e.g. according to ISO-31000 (2018)), then it is either the description of a potential effect (effect of uncertainty on objectives) or a measure of it (frequency and consequence). Risk does not specify a goal.

Reliability is a desired system property and a measure as well (survival probability). This has resilience in common with reliability, but there is no established metric. Whereas in risk and reliability analysis (undesired) events are identified and evaluated, in resilience analysis it is often only a listing of components and principles contributing to system resilience.

The differences between resilience and reliability or availability have not yet been clearly clarified on a formal level (cf. Mock et al. (2019)). Overall, resilience has fundamental similarities with the concept of ensuring business continuity.

Mock et al. (2019) showed that there are plenty and various resilience definitions used in literature. Resilience is understood as an ability, a state, a quality, or a property of a system. In this paper, resilience is considered as the property of a system to recover from a disruptive event. To be resilient, the system has to react and return

to a normal operating mode. The definition of the resilience property consists of different terms that have to be defined as well. These terms are system, disturbance, reaction, and normal operating mode. Technical systems are designed for a specific purpose. The performance characterizes the quality of the fulfillment of this purpose.

Three operating modes of the system are considered in this paper. They are the normal operating, degraded operating and collapsed mode. Startup, shutdown, or maintenance modes are not considered. So first, the normal operating mode is the regular undisturbed operation of a system under normal conditions. Second, the degraded operating mode means that additional efforts have to be made to maintain the operation. For example additional unexpected costs, business continuity management, or degraded performance. Third, the collapsed mode indicate that the system is not operating anymore and the performance is zero.

Starting from the operating mode, the events disturbance and reaction can be defined. A disturbance is defined as an event that changes operating mode from normal to degraded. A reaction is defined as an event that is triggered by the system but is not part of the normal operation, it changes the operating mode from degraded to normal.

In order to embed resilience in a broader context, notions of stability and instability are needed. Stability is the property to withstand an event and to stay in a normal operating mode. Instability is the property of not being able to withstand an event, so neither staying operational nor degraded operational. So for a given system and an occurring event, the system has one of the following properties: stability, resilience, or instability.

So for a system designer, the question is which system property is desired for a set of possible events. Ideally, the system is stable for all possible events. But such a requirement is unrealistic and would need a lot of resources. Therefore, it make sense to settle for stability only for high frequent events. For moderately frequent events resilience is desired. For low frequent events instability can be tolerable. The acceptable bounds between high, moderate, and low frequent events are strongly dependent on the analyzed system.

This approach shows similarities with the frequency-consequence concept of risk. The assessment of the risk parameters use an ordinal scale. To handle risks, system designer implement the defence-in-depth strategy. The idea is to handle disruption or risks along different defence lines. The first defense line is that most disruptions are tried to be prevented by proactive secure design of the system and by use of high quality standards and implementations. The next defense lines are to handle occurring disruptions to minimize and avert damage, cf. International Nuclear Safety Advisory Group (1999).

If a system designer constructs a secure system

by implementing defence-in-depth, the system will show for events as consequences the system properties as depicted in Table 1. In concrete terms, this means frequent events have no consequences. Moderate frequent events can be averted and performance losses minimized. Massive disruptions, that are beyond the design, can lead to instabilities.

Table 1. System property for different frequent events in case of implemented defence-in-depth.

Event frequency	Consequence: system property
high	stability
moderate	resilience
low	instability

The quality of the system design and the assumptions of Table 1 has to be validated. For this purpose, the frequency of an event is relatively easy to obtain. But the impact and consequences such as the according property of the system is hard to obtain. Therefore it is suggested to model and simulate the system for high and moderate frequent events.

### 3. Quantitative resilience analysis method (QReAM)

In this section, a method is introduced to analyze the stability, resilience, and instability properties of a system. This method is referred to as quantitative resilience analysis method (QReAM) in the following. Although this method is applicable for different types of systems. This paper focusses on power grids.

The power grid includes transmission lines, electrical generators, and electric power demands. For events, fluctuating power demands are taken into account. A possible reaction consists in load dispatching, i.e. the demand requirements do not necessarily have to be met.

The power grid system is described by a direct current (DC) power flow problem. This problem can be derived using assumptions that the power grid is in a steady state, the frequency is constant, losses and reactive power can be neglected, and voltage magnitudes are constant, cf. Bergen and Vittal (2000). Then, the grid is modeled by busses and branches, where busses contains generators and demands and the branches model transformers and transmission lines.

The set of buses is denoted by  $\mathcal{S}_B = \{1, \dots, N_{bus}\}$ . Each bus  $i$  is associated with an voltage angle  $\varphi_i$ , which is fixed at a reference bus  $\varphi_{ref} = 0$ . The power demand  $P_{D_i}$  at bus  $i$  is prescribed.  $\mathcal{S}_G \subset \mathcal{S}_{bus}$  is the set of buses with a generator, where the generated power  $P_{G_i}$

variable. The cost of power generation is denoted by  $c_i(P_{G_i})$ . Furthermore the generated power is bounded from below and from above ( $P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max}$ ). If a bus is not in the set of generators, it implies that  $P_{G_i} = 0$ .

There is also a set of branches  $\mathcal{L}$ . Each branch is associated with the susceptance matrix entry  $\mathcal{B}_{ij}$ . If the buses  $i$  and  $j$  are not connected, it implies that  $\mathcal{B}_{ij} = 0$ . The power flow between two buses is computed with the angle difference  $P_{ij} = \mathcal{B}_{ij}(\varphi_i - \varphi_j)$ . As the power can not disappear and has to be constant for one bus, the DC power flow equations are written as

$$P_{G_i} - P_{D_i} = \sum_{j \in \mathcal{S}_B} P_{ij} \text{ for } i \in \mathcal{S}_B. \quad (1)$$

Furthermore, there are constraints that have to be satisfied on the branches. The angle difference on a branch is not allowed to exceed a certain maximum  $|\varphi_i - \varphi_j| \leq \varphi_{ij}^{\max}$ .

This leads to the DC approximation of the optimal power flow problem:

$$\text{minimize } \sum_{i \in \mathcal{S}_G} c_i(P_{G_i}) \quad (2)$$

subject to

$$\begin{aligned} P_{G_i} - P_{D_i} &= \sum_{j \in \mathcal{S}_B} P_{ij} \text{ for } i \in \mathcal{S}_B, \\ P_{G_i}^{\min} &\leq P_{G_i} \leq P_{G_i}^{\max} \text{ for } i \in \mathcal{S}_G, \\ |\varphi_i - \varphi_j| &< \varphi_{ij}^{\max} \text{ for } \{i, j\} \in \mathcal{L}, \\ \varphi_{ref} &= 0. \end{aligned}$$

The dispatchable loads are modeled by converting power demands into generators. The buses with dispatchable loads  $P'_{D_i}$  are in the set  $\mathcal{S}_D$ . This means that the dispatchable power demand  $P'_{D_i}$  becomes variable and is bounded to its previous demand value  $P_{D_i}$ . The additional objective functions  $l_i(P'_{D_i}) < 0$  for the dispatchable loads are introduced. This function denotes the loss in revenues, which should be minimized. This leads to the following modified problem:

$$\text{minimize } \sum_{i \in \mathcal{S}_G} c_i(P_{G_i}) + \sum_{i \in \mathcal{S}_D} l_i(P'_{D_i}) \quad (3)$$

subject to

$$\begin{aligned}
 P_{G_i} - P_{D_i} &= \sum_{j \in \mathcal{S}_B} P_{ij} \text{ for } i \in \mathcal{S}_B \setminus \mathcal{S}_D, \\
 P_{G_i} - P'_{D_i} &= \sum_{j \in \mathcal{S}_B} P_{ij} \text{ for } i \in \mathcal{S}_D, \\
 P_{G_i}^{\min} &\leq P_{G_i} \leq P_{G_i}^{\max} \text{ for } i \in \mathcal{S}_G, \\
 0 &\leq P'_{D_i} \leq P_{D_i} \text{ for } i \in \mathcal{S}_D, \\
 |\varphi_i - \varphi_j| &< \varphi_{ij}^{\max} \text{ for } \{i, j\} \in \mathcal{L}, \\
 \varphi_{ref} &= 0.
 \end{aligned}$$

The solvers of MATPOWER of Zimmerman et al. (2011) are used for the DC optimal power flow problems (2) and (3).

In the next step, three time dependent functions are introduced:  $operational(t)$ ,  $degraded(t)$ ,  $collapsed(t)$ . They indicate the operating modes of the system in time. If the function value is one the system is in this operating mode, otherwise it is zero. Note that the system can only be in one and only one of the three operating modes at the same time.

To determine these functions, the power grid is simulated over a time interval  $[0, T]$  with  $N_T$  time steps. The power demands are changing at every time step  $t_i$ [h]. If there is a feasible solution to the problem (2), the system is in full operational mode, which means

$$\begin{aligned}
 operational(t_i) &\leftarrow 1, \\
 degraded(t_i) &\leftarrow 0, \\
 collapsed(t_i) &\leftarrow 0.
 \end{aligned} \tag{4}$$

In cases where the optimal power flow solver cannot find a feasible solution, the modified power flow problem with load dispatching (3) is solved. If this modified power flow problem has a feasible solution, the system is then considered to be in a degraded operation mode

$$\begin{aligned}
 operational(t_i) &\leftarrow 0, \\
 degraded(t_i) &\leftarrow 1, \\
 collapsed(t_i) &\leftarrow 0.
 \end{aligned} \tag{5}$$

If there is no feasible solution for both problems, the system is considered as being in collapsed operation mode

$$\begin{aligned}
 operational(t_i) &\leftarrow 0, \\
 degraded(t_i) &\leftarrow 0, \\
 collapsed(t_i) &\leftarrow 1.
 \end{aligned} \tag{6}$$

The method to determine the operational modes over time is summarized in Algorithm 1.

From the determined operational mode functions, the system properties as stability, resilience,

**Algorithm 1:** Method to analyze the operational mode for varying events.

```

for  $t_i$  in  $[0, \dots, T]$  do
  feasible = solveOPF( $t_i$ );
  if feasible then
    operational( $t_i$ )  $\leftarrow$  1;
    degraded( $t_i$ )  $\leftarrow$  0;
    collapsed( $t_i$ )  $\leftarrow$  0;
  else
    feasible = solveModifiedOPF( $t_i$ );
    if feasible then
      operational( $t_i$ )  $\leftarrow$  0;
      degraded( $t_i$ )  $\leftarrow$  1;
      collapsed( $t_i$ )  $\leftarrow$  0;
    else
      operational( $t_i$ )  $\leftarrow$  0;
      degraded( $t_i$ )  $\leftarrow$  0;
      collapsed( $t_i$ )  $\leftarrow$  1;
    end
  end
end
  
```

and instability can be derived. If the operation mode is fully operational for this time frame, the system is stable. In case the system gets into a collapsed operating mode the system is instable. In case the system gets into a degraded operational mode from time to time but stays operational the rest of the time, the system has the property of resilience.

To get a more precise notion of the system properties, long time frames are considered. The averages of the operating mode functions are defined by

$$\begin{aligned}
 \overline{operational} &= \frac{1}{N_T} \sum_{i=1}^{N_t} operational(t_i), \\
 \overline{degraded} &= \frac{1}{N_T} \sum_{i=1}^{N_t} degraded(t_i), \\
 \overline{collapsed} &= \frac{1}{N_T} \sum_{i=1}^{N_t} collapsed(t_i).
 \end{aligned} \tag{7}$$

From these values, the system properties can be classified.

- The system is stable if  $\overline{operational}$  is close to one and  $\overline{degraded}$  as well as  $\overline{collapsed}$  are neglectable small.
- The system is resilient if  $\overline{operational}$  and  $\overline{degraded}$  are less than one and  $\overline{collapsed}$  is less than a small constant  $\varepsilon$ .
- The system is instable if  $\overline{operational}$  and  $\overline{degraded}$  are less than one and  $\overline{collapsed}$  is greater than a small constant  $\varepsilon$ .

The constant  $\varepsilon$  is an input parameter, which has to be determined by the user. Ideally, it is set to a very small value just to accommodate for rare events. But in practice larger sizes are advised depending on the requirements of the system. The classification of the system properties is summarized in Table 2.

Table 2. Classification of system property.

System property	Condition
stability	$1 - \varepsilon \leq \overline{\text{operational}} \leq 1$ , $0 \leq \overline{\text{degraded}} \leq \varepsilon$ , and $0 \leq \overline{\text{collapsed}} \leq \varepsilon$ .
resilience	$0 \leq \overline{\text{operational}} < 1 - \varepsilon$ , $0 \leq \overline{\text{degraded}} \leq 1$ , and $0 \leq \overline{\text{collapsed}} \leq \varepsilon$ .
instability	$0 \leq \overline{\text{operational}} < 1 - \varepsilon$ , $0 \leq \overline{\text{degraded}} < 1$ , and $\varepsilon < \overline{\text{collapsed}} \leq 1$ .

In the context of stability and instability further quantification is not needed, but in the context of resilience quantification is desired. Two resilience measures are introduced here. It can be done either by using the average of operational mode function or by using an averaged performance indicator. It indicates that for an event the system stays longer in full operating mode and the times of degraded operation are shorter. The met demands over the total demand is proposed as time dependent performance indicator, which is written as

$$p(t_i) = \frac{\sum_{i \in S_B \setminus S_D} P_{D_i} + \sum_{i \in S_D} P'_{D_i}}{\sum_{i \in S_B} P_{D_i}}. \quad (8)$$

Note that this performance indicator cannot exceed one or fall below zero. The averaged operational mode function is easier to obtain and relatively robust. The performance indicator in general may not be easily obtained or defined, but can give further insights.

#### 4. Case study

In this section, the QReAM and measures introduced in the previous Section 3 are applied to a small distribution grid as introduced in Baran and Wu (1989). In Figure 1, this grid is visualized using the Thevenin impedance distance as described in Cuffe and Keane (2017). The grid contains 33 buses and 37 branches. One of the buses is a generator, the rest are demand buses. The power demand  $P_{D_i}$  of the loads is lognormal distributed, with a mean of 0.187 MW and a varying variance  $\sigma^2$  [MW]. The generator cost for bus

one is  $c_1(P_{G_1}) = P_{G_1} \cdot 20$  \$/MWh. The limits are  $P_{G_1}^{\min} = 0$  MW and  $P_{G_1}^{\max} = 10$  MW. The limit of the angle differences is for all branches the same  $\varphi^{\max} = 360^\circ$ .

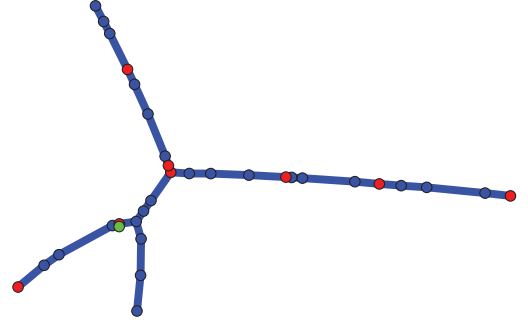


Fig. 1. Small distribution grid with 33 buses. The blue, green, and red dots denote load buses, generator buses, and dispatchable load buses, respectively. The blue lines correspond to transmission lines. The distances correspond to the Thevenin impedance.

As mentioned before, load dispatching is used as reaction. Three scenarios are considered in the following. The set of dispatchable load buses has either four, eight, or 16 elements. This corresponds to around 12, 24, and 48 percent of the buses.

In a first experiment, the system with 24 percent dispatchable loads is used. The operating mode is studied over time for three variances, namely  $10^{-4}$  MW,  $10^{-2}$  MW, and  $10^{-1}$  MW. The number of time steps is 20 h.

In Figure 2, it can be seen that for  $\sigma^2 = 10^{-4}$  MW, the system is always operational and thus is deemed stable according to Table 2 in this time interval. For  $\sigma^2 = 10^{-2}$  MW, the system gets into a degenerated operating mode from time to time but is never collapsing, so it is deemed resilient in this time frame. For  $\sigma^2 = 10^{-1}$  MW, the system gets into a collapsed mode from time to time, so the system is unstable in this time frame.

In Figure 3, the performance indicator is shown for the same time series. The collapses coincide with the operating mode analysis, but the distinction between degraded and operational is not as sharp as for the operating mode analysis.

The QReAM is used to quantify the properties of the system more precisely. So first, the averages of the operating modes (7) are computed. For this, a longer time interval is used, namely 800 h. This is done for variances from  $10^{-4}$  MW up to  $10^{-1}$  MW. The systems with 12, 24, and 48 percent dispatchable load buses are shown in Figure 4. There, for small variances ( $\sigma^2 \leq 10^{-3}$  MW)



the three systems are operational all the time and are therefore stable. For moderate variances ( $10^{-3} \text{ MW} \leq \sigma^2 \leq 10^{-2} \text{ MW}$ ), all three cases lose a portion of the operational mode and thus increase the portion of degraded mode. For high variances ( $10^{-2} \text{ MW} \leq \sigma^2$ ), the systems start to differ in the increase of collapsed fraction. The more loads can be dispatched the more collapses can be avoided.

To get from the averaged operating modes to the system properties Table 2 is used. The parameter  $\varepsilon$  is set to  $10^{-2}$ . The classification is shown in Figure 5. It can be seen that the area of stability is the same for all three cases. But the area of resilience is increasing with the number of dispatchable loads, which means that the area of instability is decreasing for higher variances.

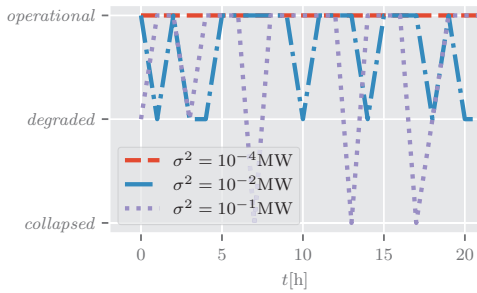


Fig. 2. Time series of operating modes for 24 percent dispatchable loads and three variances. The variances  $10^{-4} \text{ MW}$ ,  $10^{-2} \text{ MW}$ , and  $10^{-1} \text{ MW}$  are denoted by the dashed, dashed-dotted, and dotted line, respectively.

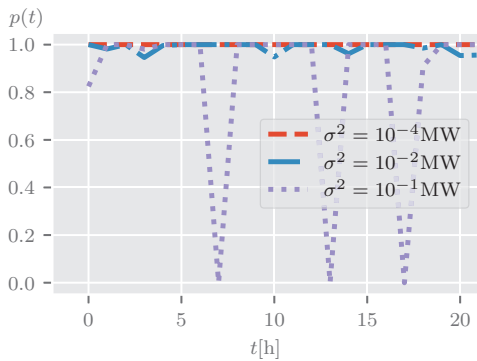


Fig. 3. Time series of performance indicator (8) for 24 percent dispatchable loads and three variances. The variances  $10^{-4} \text{ MW}$ ,  $10^{-2} \text{ MW}$ , and  $10^{-1} \text{ MW}$  are denoted by the dashed, dashed-dotted, and dotted line, respectively.

At last resilience is quantified. Figure 6 shows the previous introduced qualifier of resilience from the operating mode. The three systems show the same resilience values when they are resilient. In Figure 7, the performance indicator is used for comparison. It turns out that the systems with more dispatchable loads are more resilient.

This case study illustrates the concept developed in this paper. It shows how the property of stability, resilience, or instability can be attributed by analyzing different operating modes. Furthermore, it illustrates how resilience then can be measured. These illustrations are complemented by a parameter study for multiple variances of power demand and different numbers of dispatchable loads. As a result, it could be shown that for small variances the power grid is stable and that for higher variances the power grid is more

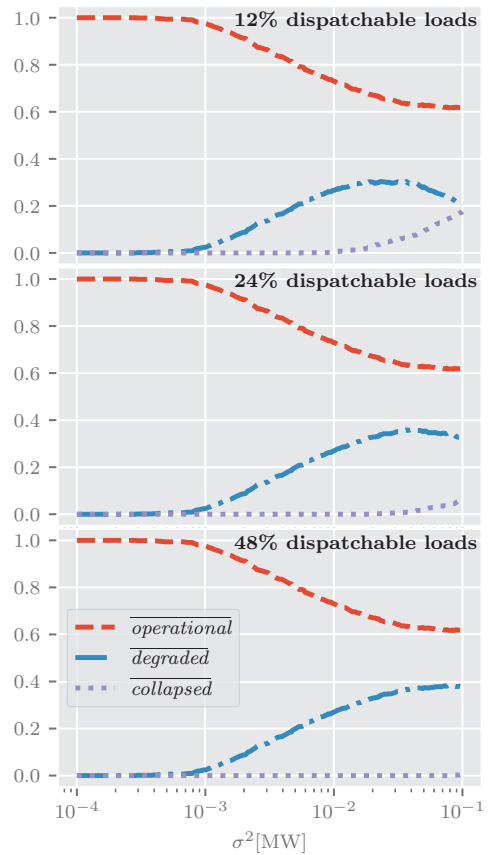


Fig. 4. The averages of operational modes with 12, 24, and 48 percent dispatchable loads from top to bottom. The average of operational, degraded, and collapsed are denoted by the dashed, dashed-dotted, and dotted line, respectively.

resilient the more loads are dispatchable.

### 5. Conclusion

In this work, a new foundation has been build to define resilience. This resilience is considered as an ability towards individual isolated events or disruptions. The resilience definition is based

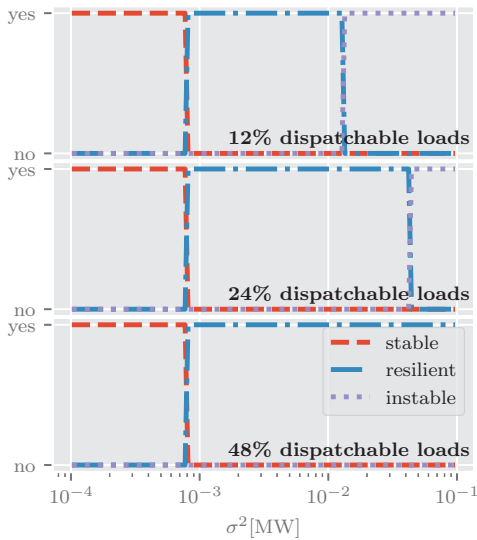


Fig. 5. Classification of system property for the tree systems with 12, 24, and 48 percent dispatchable loads from top to bottom. The system property stable, resilient, and instable are denoted by the dashed, dashed-dotted, and dotted line, respectively.

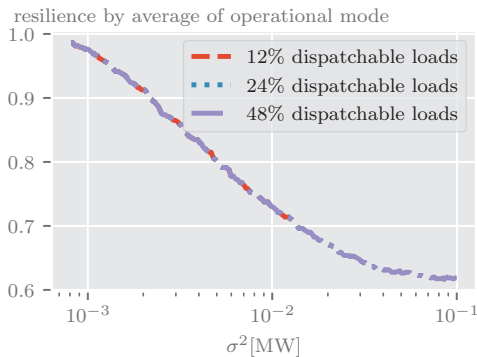


Fig. 6. Averaged operating mode as resilience measure in the case of resilience. The three systems with 12, 24, and 48 percent dispatchable loads are denoted by the dashed, dashed-dotted, and dotted line, respectively.

on well established terms, such as disruption, reaction, normal operating mode, degraded operating mode, and collapsed. This is in contrast with many other definition that are based on performance indicators. So, the resilience ability, presented in this work, is strongly dependent of the system bound between normal operation and reaction. In the authors' opinion, this boundary can be drawn more sharply than the fluctuations of the performance indicator can be distinguished. On top of this definition resilience is brought into context with stability and instability. These definitions already allow qualitative assessments of resilience, which can be used for planing or operating purpose of systems in general. This general approach also means that some details are neglected such as disruption impact or time to reaction.

Based on this approach, the new QReAM has been developed. QReAM classifies event as stable, resilient, or instable. This classification is strongly associated with the system and its reaction. QReAM can be easily adapted to many systems and events, but has been presented here for power grids. The power grid is modeled by computing the actual power flow. To capture the stochastic power demand, the analysis considers the average in of the operating modes for the classification of the system ability. The paper only models the fluctuations of normal grid operation. Nevertheless they can cause disruptions. The considerations of incidents and failures beyond normal operation is subjected of the ongoing research project. However, these fluctuations can be seen as a base on top of which all other incidents (e.g. generator failure) have to be tested against. In concrete terms, it is not enough to show that the power grid is stable or resilient at a summer

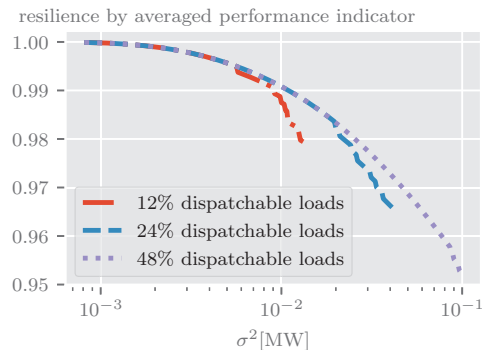


Fig. 7. Averaged performance indicator as resilience measure in the case of resilience. The three systems with 12, 24, and 48 percent dispatchable loads are denoted by the dashed, dashed-dotted, and dotted line, respectively.

vacation day, but should be also stable or resilient at winter working day. QReAM can be relative easily be implemented and has polynomial complexity.

In this work, QReAM has been used for a case study. Despite the small grid size and limited complexity, conclusions can be drawn. It could be shown that the stability and the resilience depends on the variance of the fluctuating power demands. Furthermore the resilience can be improved by increasing the number of dispatchable loads. This lays a strong foundation for extensions and future work.

In future work, QReAM might incorporate further analyses. For example, it could be of interest to analyze the reason for degraded or collapsed performance. These reasons can be either due to the limitations of the generator or due to the limitation of the transmission line. If a specific transmission line or generator is a main causes for degraded or collapsed performance, the method could make suggestion how to improve the system design.

Additionally, the case study could be investigated further. It could be of interest to analyze the influence of the grid topology to the resilience. Also, it could be analyzed how the location of the dispatchable loads affects resilience. More events could be considered, such as potentially machine failures, human errors, or natural disasters. This means that also further reactions have to be considered, such as reparation or power imports.

Having covered most of the possible events, it could be aimed at a resilient measure independent of specific events. The power grid will be replaced with larger and more realistic ones. Also the stochastic demands will be replaced with more realistic load profiles capturing different demand types and different time cycles such as seasonal, week, and day cycles. The main contribution of this paper is a novel definition of the resilience ability and the development of QReAM which allows to quantify this ability.

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