

Integrating component condition in long-term power system reliability analysis

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The electric power system is a critical infrastructure in which power transformers play a key role in linking together generation and end-use of electricity. The consequences of transformer breakdown can be significant, and aging transformers have a higher probability of failure. For decisions in asset management and power system development, it will therefore be useful to capture how deteriorating component condition affects failure probabilities and the overall reliability of the power system. Since such decisions have planning horizons of multiple years, the analysis should also capture similar time horizons. To this end, this paper proposes an analytical approach to power system reliability analysis (PSRA) accounting for time dependencies in the technical condition of components. An analytical PSRA methodology integrating a transformer condition model is extended to analysis horizons of multiple years. This analytical methodology is compared with a Monte Carlo simulation (MCS) approach to PSRA by applying both to a realistic case study. The comparison validates the analytical approach by showing that the inaccuracies its approximations introduce are negligible, at least for the considered case. This means that the proposed methodology can be a computationally viable alternative to MCS methods, especially when it is too time consuming to assess the impact of different scenarios with sufficient statistical precision using MCS. However, drawbacks with the analytical approach for further extensions of the methodology are also discussed.

Keywords: Degradation, Maintenance engineering, Power system reliability, Power transformer, Monte Carlo simulation

1. Introduction

Aging power systems with deteriorating components is a major concern for the continued reliable supply of electric power. Addressing this concern calls for an integrated approach to power system reliability analysis in which reliability analyses both at component level and system level are included. Traditionally, the first level focuses on a single asset or component in the power system (e.g., a transformer station) but does not properly account for its importance in the power system for the reliability of supply. The second level takes a broader view of the power system but usually neglects how the condition of individual components influences their probability of failure and how this contributes to the overall power system risk. For instance, reliability of supply analyses applied for long-term power system planning studies commonly assume the same failure rate for all components of the same type. However, it is well known that deteriorated power system

components have a higher probability of failure than new components.

The overall aim of this work is to account for the technical condition of power system components (such as transformers) in long-term power system reliability analyses to better inform power system development and asset management decisions. The work presented in this paper builds upon previous research integrating a transformer health model with an existing, analytical power system reliability analysis (Toftaker et al., 2022). In this modelling framework, condition is modelled by a health index, and probability of failure is given by a lifetime distribution evaluated at the corresponding apparent age. Employing an analytical approach to the power system reliability analysis has benefits in terms of computational efficiency and analytical transparency. However, a drawback with the previously developed analytical approach is that it is applicable for an analysis horizon of around just one year into the future.

This paper extends the work by Toftaker et al. (2022) to a longer time horizon by modelling how technical condition develops over time, both due to deterioration and maintenance or replacement. The extension is important to make the methodology applicable to decision making, for instance by assessing the long-term benefit of replacing an old transformer with a new one.

The rest of this paper is structured as follows. Section 2 introduces the necessary theoretical background for power system reliability analysis in general and the specific analytical methodology that is employed in particular. To validate the extended analytical methodology proposed in this paper, a Monte Carlo simulation (MCS) approach is employed, and Section 2 therefore also introduces the fundamentals of MCS approaches to power system reliability analysis. Section 3 first summarizes the component reliability model of Toftaker et al. (2022) before presenting the proposed extensions to longer time horizons. In Section 4, the analytical and MCS-based approach to accounting for time dependencies in component condition are applied to a case study. Finally, a summary of the comparison between the two approaches and suggestions for further extensions of the methodologies is given in Section 5.

2. Power system reliability analysis

Methods for power system reliability analysis can be broadly divided into two groups: i) analytical methods and ii) Monte Carlo simulation methods (Li, 2014; Billinton and Allan, 1996; Billinton and Li, 1994). In this paper we both propose an extension of the analytical method presented in Toftaker et al. (2022) and validate it using a MCS method. Therefore this section first introduces the theoretical background necessary for both approaches to evaluating the reliability of a power system, before specifying the analytical method in Section 2.1 and the specific MCS method used in Section 2.2.

Reliability of supply analyses are concerned with the electricity supply at delivery points or load points in the power system. The results are the values of a set of reliability of supply indices for a set of delivery points, with the annual energy not supplied being an important example of

such a reliability index. It measures the long-term average ability of the power system to provide electric power to end-users. At a given point in time t , this ability is determined by the system state that we describe as a combination of the contingency state and the operating state. The contingency state is the combination of the functional state of the individual power system components. This contingency state may be represented by a binary vector $\mathbf{V}(t) = [V_1(t), \dots, V_n(t)]$, where $V_1(t) = 1$ denotes that component 1 is in service and $V_1(t) = 0$ denotes that it is in an outage state, etc. The operating state is characterized by the load and generation composition in the system. For the purposes of this paper, it is described by the load demand at all the load buses (delivery points) $\mathbf{P}(t) = [P_1(t), \dots, P_m(t)]$.

To evaluate the energy not supplied it is necessary to estimate the amount of power supply interruption (or loss of load) for each system state that is considered in the reliability analysis. This is done using a contingency analysis. In this work we will employ the contingency analysis implementation of Gjerde et al. (2016) both for the analytical and MCS-based reliability analysis. It is based on solving an optimal power flow problem to obtain an estimate of the system available capacity (SAC) for each delivery point. Each combination of operating state and contingency state included by the reliability analysis methodology is then evaluated to obtain SAC as a function of time for each delivery point k . The interrupted power at time t and delivery point k for contingency j is given by

$$P_{\text{interr},j,k}(t) = P_k(t) - \text{SAC}_{j,k}(t) \quad (1)$$

2.1. Analytical approach

Most analytical reliability analysis methods are characterized by considering a pre-defined set of contingencies rather than sampling contingencies randomly as in a MCS methods. This is called the contingency enumeration approach, and the contingency set can be specified to include, e.g., all first- and second-order component outages. Other typical limitations of analytical methods are that they are based on calculating expected values assuming that the underlying stochastic processes have reached a steady state. It can be cumbersome

to include time dependencies, and it was primarily this limitation that motivated the work presented in this paper.

The type of analytical, contingency enumeration method that we consider here is the minimal cut set method (Billinton and Allan, 1996), where contributions to reliability of supply indices are calculated for each contingency j that corresponds to a minimal cut set for the power supply to delivery point k . More specifically, as Toftaker et al. (2022) we employ an implementation of the OPAL methodology (Kjølle and Gjerde, 2012; Gjerde et al., 2016). OPAL extends the standard minimal cut set methodology by, among other things, accounting for multiple operating states. The analysis horizon of one year is divided in a discrete set of operating states \mathbf{P}_o , and each operating state is associated with a set of hours of the full years. The hours associated with each operating state do not need to be consecutive.

This analytical reliability analysis methodology rests on the assumption that the reliability of individual power system components are described by a two-state Markov model. This implies that the underlying probability distributions for the time to failure (or in general transitions between the two states) are exponential, and consequently, that the components' failure rate functions are constant in time.

Each reliability analysis then considers all combinations of a pre-defined set of operating states and a pre-defined contingency list. The SAC is evaluated for all delivery points for each combination of the operating state o and contingency j to obtain the interrupted power $P_{\text{interr},o,j,k}$. According to this analytical power system reliability analysis method, contributions to the annual expected energy not supplied (EENS) are calculated as

$$\text{EENS}_{a,o,j,k} = \lambda_{o,j} \cdot r_{o,j} \cdot P_{\text{interr},o,j,k}, \quad (2)$$

where $\lambda_{o,j}$ and $r_{o,j}$ are the equivalent failure rates and outage times for contingency j and operating state o . These contributions are then aggregated to obtain the annual expected energy not supplied for a delivery point by summing over the set of operating states and set of contingencies.

2.2. Sequential Monte Carlo simulation

Another approach to evaluate the reliability of the power system is by Monte Carlo simulation. For an introduction to Monte Carlo sampling applied to power systems the reader is referred to Billinton and Li (1994). We propose a sequential Monte Carlo algorithm to capture time dependence of the condition-dependent probability of failure. The idea is to draw random samples of the contingency state of the power system as a function of time. This is done by a state duration sampling technique for sampling the functional state of each individual component of the system as a function of time. This choice is made to allow for general probability distributions for the time to failure (i.e., not limited to exponential distributions) in a flexible manner.

The result of the state duration sampling is a time series of contingency states for each Monte Carlo sample i , $\mathbf{V}_i(t)$. We use the same discretization of the time horizon into operating states as above. Each unique combination of operating state and contingency state is evaluated to obtain the interrupted power as a function of time for each simulation $P_{\text{interr},k,i}(t)$.

The energy not supplied is given as an integral over time, which is evaluated as a sum across hours within the analysis horizon

$$\text{ENS}_{k,i} = \sum_{t=1}^T P_{\text{interr},k,i}(t) \quad (3)$$

A Monte Carlo estimate of the expected energy not supplied is given by the mean across the Monte Carlo samples

$$\text{EENS}_k = \sum_{i=1}^N \frac{\text{ENS}_{k,i}}{N} \quad (4)$$

3. Component reliability

To include technical condition into the reliability analysis we introduce a component reliability model. The model is similar to the one presented by Toftaker et al. (2022). It is important to note that it describes the condition of the power system component that at any time fills a certain function in the power system. It does in other words not describe the individual physical components

themselves, which may be replaced or retired. Although the model is general, it will later in the paper be applied only to transformers.

We consider a component that may fail by two independent mechanisms, where the first is referred to as wear-out failures and depends on the technical condition of the component and the other is referred to as mid-life failures and is independent of condition. Following Foros and Istad (2020), the time until wear-out failure T_w follows the probability distribution $F_{T_w}(s)$, where s is the apparent age of the component. It is assumed that if the health index at calendar age t_0 is HI_0 and the corresponding apparent age s_0 is $s_0 = \tau_0(HI_0)$. The relation τ between health index and apparent age may be obtained through statistical data as illustrated by Foros and Istad (2020). To obtain apparent age as a function of calendar time it is further assumed that apparent age increases at the same rate as calendar age, i.e. $s(t) = \tau_0(HI_0) + t$, where it is assumed that the present time is $t = 0$. The development and integration of less simplistic condition prognosis models are left for future work.

The time to mid-life failure follows an exponential distribution with rate λ_{ml} . If a failure occurs the component remains in a failed state for a time period T_R until a repair is executed. The repair time T_R is assumed to be exponentially distributed with rate μ . If a wear-out failure has occurred a replacement is required, which means condition is reset to as good as new. This in turn means that subsequently, its apparent age restarts at 0. If a mid-life failure has occurred it is assumed that a minimal repair is sufficient, and the condition remains unchanged. Furthermore, the component may be preventively retired (replaced) to reset the condition to as good as new (Toftaker et al., 2022). In the lack of more detailed models, the time until preventive replacement T_{pm} is assumed to be exponentially distributed with rate λ_{pm} .

In consequence the functional state of the component follows a semi-Markov model as illustrated in Figure 1. This semi-Markov model was by Toftaker et al. (2022) simplified to a Markov model as illustrated in Figure 2. This was achieved by two assumptions: First, that within the analysis

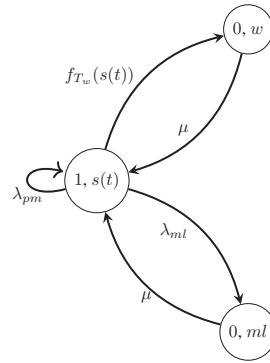


Fig. 1. A diagram illustrating the probabilistic failure model.

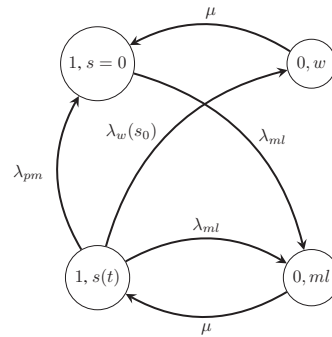


Fig. 2. A diagram illustrating the simplified probabilistic failure model.

horizon the technical condition does not change significantly, so that the time to wear-out failure is exponentially distributed with a rate

$$\lambda_w(s_0) = \frac{F_{T_w}(s_0 + 1) - F_{T_w}(s_0)}{1 - F_{T_w}(s_0)} \quad (5)$$

Second, that after the component is replaced, the probability of wear out failure is negligible. Let N_{t_1, t_2} denote the number of failures within the time period t_1 to t_2 . It can be derived (Toftaker et al., 2022) that, with $\lambda_w = \lambda_w(s_0)$, the expected number of wear out failures within the next year, is given by

$$\mathbb{E}(N_{0,1}) = \frac{\lambda_w}{\lambda_w + \lambda_{pm}} (1 - e^{-(\lambda_w + \lambda_{pm})}). \quad (6)$$

3.1. Extended time horizon

To extend the component reliability model to a longer time horizon we propose a recursive

scheme. First, we recognize that the expected number of wear-out failures for component j in year t is given by the law of total expectation as

$$\mathbb{E}(N_{t,t+1}) = \sum_{s=0}^{\infty} \mathbb{E}(N_{t,t+1}|S_t = s)P(S_t = s)$$

where S_t is the apparent age of the component at the end of year t , and $\mathbb{E}(N_{t,t+1}|S_t = s)$ is given by (6) with $\lambda_w = \lambda_w(s)$. We introduce the time-dependent failure frequency $\omega_{w,t}$ to denote the expected number of wear-out failures in year t .

Figure 3 illustrates the process determining the development of the apparent age S of a power system component. The apparent age of a component at the present time is denoted s_0 , and we have that $P(S_0 = s_0) = 1$.

The recursive expression for the probability of having a certain apparent age at the beginning of year t is then

$$P(S_t = s) = \sum_{s'=0}^{\infty} P(S_{t-1} = s')Q_{s',s},$$

where $Q_{s',s} = P(S_{t+1} = s|S_t = s')$. Exploiting the fact that during year t the component can either become 1 year older or replaced it can be derived that, for $t > 0$,

$$P(S_t = 0) = \sum_{t'=0}^t [P(S_{t-1} = t')Q_{t',0} + P(S_{t-1} = s_0 + t')Q_{s_0+t',0}],$$

and for $s \neq 0$,

$$P(S_t = s) = P(S_{t-1} = s - 1)Q_{s-1,s}.$$

This calculation procedure is illustrated up to $t = 2$ as an event tree in Figure 3.

The probability to transition from apparent age s to apparent age 0 within the next year from time t to time $t + 1$:

$$Q_{s,0} = P(S_{t+1} = 0|S_t = s) = 1 - e^{-(\lambda_{pm} + \lambda_w(s))}.$$

The probability to transition from apparent age s to apparent age $s + 1$ within the next year from time t to time $t + 1$:

$$Q_{s,s+1} = 1 - Q_{s,0}$$

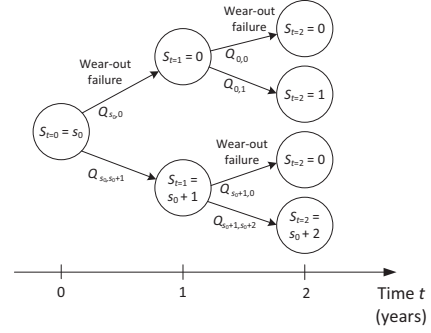


Fig. 3. A simplified event tree illustrating how the apparent age S_t of a power system component develops with time t .

Finally, a overall time-dependent failure rate λ_t for the transformer, considering mid-life failures as well as wear-out failures, is derived from $\omega_{w,t}$ using the equations presented in Sec. III.C and III.D of Toftaker et al. (2022). This failure rate can then be used as input data to the analytical power system reliability analysis to estimate annual reliability indices. In this way, each of the years of the analysis horizon can be evaluated independently by the power system reliability analysis. If the operating states are assumed to be identical for each of the years, the contingency analysis only has to be carried out for one of the years, and the results for the interrupted power can be reused for all the other years.

3.2. Sequential Monte Carlo simulation integrating condition dependence

The component reliability model in Section 3 is also integrated in a sequential Monte Carlo simulation. This section presents an algorithm to sample the functional state of a component following the process illustrated in Figure 1. Assuming that the component starts out in an in-service state, the time of the next transition is equal to the minimum of the 3 latent times, T_{pm} , T_w , T_{ml} . The algorithm therefore samples the times from their respective distributions. T_{ml} and T_{pm} are sampled from an exponential distribution. A sample of T_w is obtained by rejection sampling (Wells et al., 2004) where a proposal value s'_w is sampled from

$F_{T_w}(s)$ and then accepted if $s'_w > s_0$. It is assumed that an efficient procedure is available to sample from $F_{T_w}(s)$, which is the case if $F_{T_w}(s)$ is from a standard class of distributions like the Normal or Weibull distributions. If a preventive replacement has happened the condition of the component is set to as good as new, i.e., $s_0 = 0$. If a failure has occurred the algorithm continues by simulating the time to repair $T_r \sim Exp(\mu)$. When a repair is done (or the component is replaced in case of a wear-out failure), the condition of the component is updated. In the case of wear-out failure apparent age is set to 0 while in the case of mid-life failure s_0 is the same as it was immediately before the failure occurred.

4. Case studies

To illustrate the proposed methodology and validate it against a MCS method, a case study is carried out based on the one presented by Foros et al. (2022). To demonstrate how the framework can be applied to support decisions in asset management two scenarios, scenario 0 and scenario 1, were analyzed. Scenario 0 represent no planned renewal, while scenario 1 represent renewal of the transformer with the worst condition.

The test system considered for the case study, displayed in Figure 4, is the same 25-bus test network considered by Foros et al. (2022), and we refer to Foros et al. (2022) and Sperstad et al. (2020) for more details. Data for the 208 operating states used in this case study are given in Foros et al. (2022). The same operating states are assumed to apply for all years of the analysis horizon. The system includes eight power transformers for which condition-dependent failures will be integrated in power system reliability analysis in the following. Where not otherwise stated, input data from Sperstad et al. (2020) are used in the case study.

We evaluate the reliability of the test system by estimating the energy not supplied for a 5 year period. Scenario 0 was evaluated using both the analytical approach described in Section 2.1 and the Monte Carlo approach described in Section 2.2, while scenario 0 was evaluated with the analytical approach only. To establish a transformer reli-

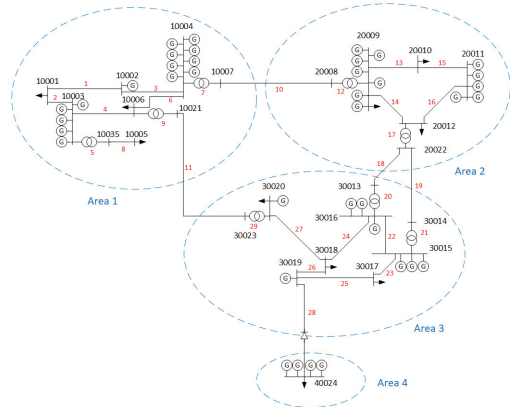


Fig. 4. Test network considered in the case study (adapted from Sperstad et al. (2020))

ability model we adopt the failure model from Foros and Istad (2020) to represent the probability distribution of time to wear out failure. Other parameters of the model are the rate of preventive replacement $\lambda_{pm} = 0.033$, the rate of mid life failures $\lambda_{ml} = 0.38$, the expected time to wear out failure $E(T_w) = 60$ years, the standard deviation of time to wear out failure $\sigma_w = 18$ years. For simplicity, scenario 3 from Foros et al. (2022), where failure rates are not calibrated to average national statistics, is considered for the case study.

The eight transformers in the test system are assigned a condition similarly as by Toftaker et al. (2022). To obtain realistic transformer condition data the set of 18 Norwegian transformers studied by Foros and Istad (2020) are used. Eight of these transformers are selected for the test system. To emphasize the importance of component condition, the transformer in worst condition has been assigned to the branch in the test network that has the biggest contribution to annual ENS (branch 29). The other transformers are arbitrarily assigned. For scenario 1 the transformer on branch 29 is replaced, which is represented by setting the health index of this transformer to 1.

4.1. Results

How the condition-dependent transformer failure frequency develop with time over the analysis horizon is shown in Figure 5. The solid curves show results for the analytical approach and the

dashed curves show results of the MCS approach. The MCS results are the estimates of the expected rate of occurrence of failures based on 6 million MC samples. For most of the transformers, the discrepancy between the results are imperceptible. The source of the small discrepancies for the transformers with $HI = 0.78$ and with $HI = 0.94$ have been identified as the rounding to integer apparent ages in the analytical approach. The main trends in the figure are i) that the failure frequency increases with decreasing condition (health index) and ii) that the failure frequency increases slightly with time. However, one can also observe iii) that for the transformer with the worst condition, the failure frequency eventually starts decreasing with time. The reason for this behaviour is that a transformer in a bad condition has a significant probability of being replaced due to wear-out failure during the next few years, after which it will be replaced by a new transformer with much lower failure rate. This effect is explicitly captured in the proposed analytical approach as well as in the MCS approach.

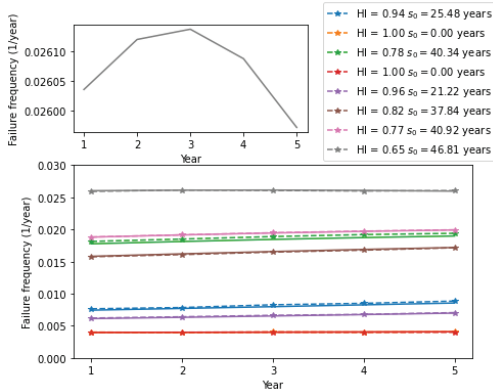


Fig. 5. Bottom: Time development of transformer failure frequencies due to degradation according to the analytical approach (solid) and the MCS approach (dashed). Top left: Failure frequency for transformer with $HI = 0.65$, with magnified y -axis.

Expected energy not supplied for each year estimated by the analytical approach and the Monte Carlo approach is shown in Figure 6. The analytical approach shows that ENS increases slightly

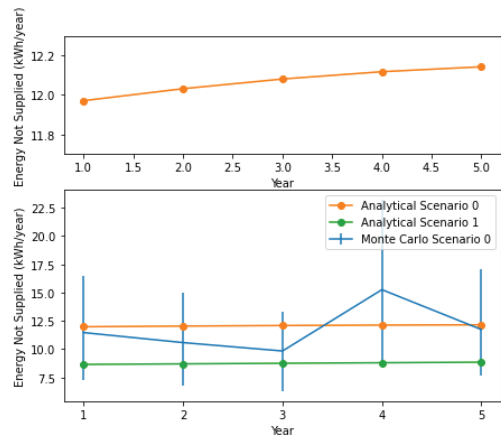


Fig. 6. Top: Annual expected energy not supplied for scenario 0 calculated by the analytical approach. Bottom: Annual expected energy not supplied. Monte Carlo results presented with 95% confidence intervals.

through the time horizon. To better show this increase the same values are plotted with a magnified y -axis in the upper plot in Figure 6. Results of the Monte Carlo method is shown with 95% confidence intervals, which are obtained by bootstrapping (Efron and Tibshirani, 1985) with 200 bootstrap samples. The Monte Carlo approach is not precise enough to show this increase. Since the width of the confidence intervals serves as a lower bound on the accuracy of the analytical approach, the MC approach does not verify the result further. It is possible to achieve better precision by using a larger sample size, but as precision is proportional to the square root of the sample size, this is not an efficient strategy to achieve significantly better precision.

Annual ENS for scenario 1 is also shown in Figure 6. Renewal of the transformer on branch 29 represents a significant reduction in annual ENS through the analysis horizon.

5. Conclusions and further work

This paper has presented an analytical approach to power system reliability analysis accounting for time dependencies in the technical condition of components. The methodology has been demonstrated on a case study accounting for power transformer condition information and has been

validated by comparing it with time-sequential Monte Carlo simulations. For the case that was considered, inaccuracies due to approximations in the analytical approach are negligible compared with the statistical uncertainty in the Monte Carlo simulation results. The changes in failure rates due to degradation over a 5-year analysis horizon are relatively small, and the resulting time dependence in the overall reliability of supply is weak. It is conceivable, however, that the importance of accounting for time-development in component condition would be greater for a case (including network and operating state data) where the degraded transformers have a stronger impact on the reliability of supply.

The results indicate that the proposed methodology can be a computationally viable alternative to MCS methods. At least, this is the case if estimating expected values are sufficient, and especially when it will be too time consuming to assess the impact of different scenarios with sufficient statistical precision using MCS. The case study considered a very reliable power system where the expected energy not supplied is very low and a relatively high number of MC iterations is needed. There is however potential for implementing variance reduction techniques to improve the computational efficiency of the MCS.

The presentation of the methodology proposed in this paper has highlighted several relevant extensions. Some of these extensions would be less cumbersome to implement in a MCS approach than in an analytical approach. First, the methodology could easily be extended to incorporate different outage times for wear-out failures and mid-life failures. If longer outage times are assumed for wear-out failures, this could reduce the accuracy of the analytical approach. In addition, a MCS approach is needed to capture the inherent variability in the outage times. A MCS approach can also more naturally integrate the simulation of asset management strategies using less simplistic models than the preventive retirement model assumed here. An interdependent direction of future work is to develop and integrate models for the time development and uncertainties in component condition over the extended analysis horizon.

Acknowledgement

This work was supported in part by the Research Council of Norway under Grant 308781 ("VulPro") and in part by Statnett, Landsnet, and Norwegian Water Resources and Energy Directorate. The authors thank all VulPro project partners for useful discussions.

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