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Optimization of Step-stress ADT following Tweedie Exponential Dispersion process

Weian Yan

School of Transportation Engineering, East China Jiaotong University, Nanchang, 330013, Jiangxi, China. E-mail: yanweian86@163.com

David Bigaud

Univ Angers, LARIS, SFR MATHSTIC, F-49000 Angers, France. E-mail: david.bigaud@ univ-angers.fr

Xiaofan Xu

School of Transportation Engineering, East China Jiaotong University, Nanchang, 330013, Jiangxi, China.

Wenqin Cao

School of Transportation Engineering, East China Jiaotong University, Nanchang, 330013, Jiangxi, China.

In this study, we focus on the optimization of step-stress accelerated degradation test (SSADT) plan when the degradation process can be modelled by a stochastic Tweedie exponential dispersion (TED) process. In the context of an optimization based on the D-optimality and V-optimality criteria, with a prior transformation of stress, the equivalence between a multilevel SSADT plan and a simple SSADT plan using only the minimum and maximum stress levels can be shown. Thus optimal SSADT plans can be simply derived. An application example will be presented to assess the effectiveness of the proposed simple optimal SSADT plans.

Keywords: Accelerated Degradation Test, Optimization, Tweedie Exponential Dispersion Process.

1. Introduction

The Accelerated Degradation Test (ADT) is a powerful tool used to assess reliability and predict the lifetime of degrading products. In a Constant-Stress ADT (CSADT), products are divided into multiple groups and exposed to a constant and severe stress condition to collect degradation data. While CSADT is an efficient test, it requires numerous units to conduct the experiment, making Step-Stress ADT (SSADT) a more appropriate choice as it requires fewer test units. Two critical tasks must be addressed to carry out the optimal design of an SSADT plan. Firstly, the optimization criterion must be selected, and secondly, a suitable degradation model must be chosen for analysing the observed degradation data. Many studies have proposed using D-optimality to improve the estimation accuracy of unknown parameters of degradation models, and V-optimality for estimating reliability indices and specific p-quantiles of lifetime distribution as optimization criteria. As for selecting the appropriate degradation model, the Tweedie exponential dispersion process (TED) process - a general class of degradation model, which includes some commonly used stochastic process (e.g., Wiener, Gamma, and Inverse Gaussian processes) as its special cases - can be used to describe more extensive degradation phenomena.

2. Model description

A stochastic process describing the evolution of a performance indicator over time is defined as an exponential dispersion (ED) process $\{Y(t),t\geq 0\}$, if satisfying the following three properties: (1) $Y(0)=0$ with probability one; (2) $\{Y(t),t\geq 0\}$ has statistically independent increments; (3) The increment follows ED distribution, i.e., $Y(t+\Delta t)-Y(t)\sim ED(\eta\Delta t,\lambda)$, for $\forall\Delta t>0$, where the probability density function (PDF) of ED distribution $ED(\mu t,\lambda)$ is expressed in Equation (1) hereafter.

$$f(y \mid t,\eta,\lambda) = c(y \mid t,\lambda) \cdot exp\{\lambda[y\omega(\eta) - t\kappa(\omega(\eta))]\} \quad (1)$$

where η is the mean drift rate, λ is the dispersion parameter; $c(\cdot)$ is a canonical function, guaranteeing that the cumulative distribution function (CDF) of Equation (1) is normalized and equal to one; $\kappa(\cdot)$ is called the cumulant function, which is a twice differentiable function, and satisfying $\kappa'(\omega(\eta)) = \eta$, in which $\kappa'(\cdot)$ is the first derivative of $\kappa(\cdot)$.

An ED model can be characterized by its variance function within the class of all ED models. Furthermore, the TED process is an important class of ED process with power variance function:

$$V(\eta) = \eta^\rho, \rho \in (-\infty, 0] \cup [1, \infty] \quad (2)$$

where ρ is the power classification parameter.

3. Optimal SSADT plans under TED process

An SSADT is characterized by the total number of test units available n, the number γ of stress levels used in the test, as well as the stress value of each level, the allocation scheme of the measurements to each stress level, the test duration τ_γ, and the measurement time interval Δt. Here, we assumed that the number of units, the test duration τ_γ, and the measurement time interval Δt are given. Therefore, the objective of the SSADT planning is to determine the optimal stress levels, as well as the proportion of units allocated to each level. The foundations of our approach are based on the following theorem demonstrated by Yan *et al.* (2023).

Theorem 1 Based on the D and V-optimality criteria, if a test unit's degradation path follows the TED process and the relationship of drift parameter and stress satisfies $\eta_k = \alpha e^{\beta s_k}$, the optimal multi-level SSADT plan using stress levels $s_1 < s_2 < \cdots < s_\gamma$ will degenerate to a simple SSADT plan using only the minimum and maximum stress levels, s_1 and s_γ.

Thus, the procedure of optimal design will follow:

Step 1: Derive the Fisher information matrix of the TED model, because many optimal SSADT design criteria are based on Fisher information matrix if the goal of conducting an experiment is to estimate the model parameters or their functions (e.g., lifetime percentiles).

Step 2: Derive that for D-optimality and V-optimality, a multi-level SSADT plan, when optimized, degenerates to a simple SSADT plan using only the minimum and maximum stress levels. This result establishes a rationale for considering a simple SSADT using only the minimum and maximum stress levels.

Step 3: We consider the design problem of a simple SSADT plan and present the optimal allocation of inspections at each stress level for different criteria.

Finally, a proof will be given that for the D-optimality and V-optimality criteria, when the total sample size n, inspection number M and the time interval Δt between inspections are given, the optimum SSADT plan based on TED process is given as follows:

(1) For **D-optimality**: the optimum plan assigns inspections: $\left(\frac{m_1}{M}, \frac{m_\gamma}{M}\right) = (p_1, p_\gamma) = (\frac{1}{2}, \frac{1}{2})$

(2) For **V-optimality**: the optimum plan assigns inspections: $\left(\frac{m_1}{M}, \frac{m_\gamma}{M}\right) = (p_1, p_\gamma) = (\frac{s_\gamma\sqrt{A_\gamma}}{s_1\sqrt{A_1}+s_\gamma\sqrt{A_\gamma}}, \frac{s_1\sqrt{A_1}}{s_1\sqrt{A_1}+s_\gamma\sqrt{A_\gamma}})$.

An application example based on actual data will be presented to compare the effectiveness of the proposed simple optimal SSADT plans and multi-level SSADT plans proposed in a previous study (Yan *et al.*, 2021). The results show that the efficiency is improved by using the optimum simple SSADT plan.

3. Major contribution and conclusion

Compared with the existing works, the major contribution of this study lies in the proof that, under the TED model with a drift parameter being an exponential function of the (transformed) stress level, a multi-level SSADT plan will degenerate to a simple SSADT plan using only the minimum and maximum stress levels under D-optimality and V-optimality criteria.

References

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