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Probabilistic Analysis of RC Doubly Curved Shells under Strong Ground Motions

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Abstract

Recently there has been a growing trend for using RC thin shell structures such as doubly curved shells (DCS) to cover large column-free spaces. However, according to the investigation of the effects of the recent earthquakes, considerable damage was observed in these structures, which negatively affected their functionality in serving as shelters during earthquakes due to the apparent lack of a general understanding of their structural response. Thus, in this study, deterministic in tandem with probabilistic analyses considering different uncertainty sources have been performed to investigate the structural response and the reliability of these structures under strong earthquakes. Due to the lack of the design of these structures in the design codes, an innovative automatic finite element-based design algorithm named the developed advanced sandwich model (DASM) is developed and employed to obtain the steel reinforcement in preparation for the analysis. The multi-axial plasticity damage constitutive model is employed through the VUMAT subroutine to reproduce the concrete nonlinear behavior. Then, a complete framework is developed to investigate the stochastic response (SR) and quantify the reliability of these structures based on the developed design procedure, concrete plasticity damage model, and the probability density evolution method (PDEM), where both the SR and the instantaneous probability density function could be attained, and a reliability of 70.78 % is obtained for the DCS under Northridge event.

Keywords: Doubly curved shells, Seismic excitation, DASM, Material nonlinearity, Nonlinear dynamic analysis, Reliability analysis.

1. Introduction

Using thin shell structures to cover large-span areas has recently been a growing trend. Also, these shell roof structures are essential to cover large span areas with high efficiency and are converted into shelters to protect peoples who lost their homes during and after earthquakes. However, shell structures have been subjected to limited research compared with other structural systems. Therefore, there is an apparent lack in the general understanding of their structural response, which negatively affects the mentioned functions of these structures, particularly in seismically active regions. Hence, an intensive investigation into this structural response to earthquakes is urgently needed, especially with the increasing threat of seismic events worldwide.

Indeed, an intensive investigation of the shells' dynamic response would need to consider several significant factors, such as reinforcement and material nonlinearity, which are rarely considered in the literature (Medwadowski SJ. 2004). Additionally, due to several uncertainty sources that significantly influence the response of these structures and are usually neglected in the deterministic analysis, the reliability of these structures cannot be quantified. The challenge of studying the stochastic response (SR) lies in different issues: designing such facilities due to the lack of design codes to obtain the steel reinforcement in preparation for the nonlinear analysis, considering the material nonlinearity through appropriate constitutive models, considering the different uncertainty sources which significantly affect the structural response, and perform the reliability analysis via an efficient method to investigates the SR and determine the structural reliability. In these regards, there is a lack of design codes for the design of these structures. However, the finite element–based design procedures were introduced as a design tool for such structures where different design procedures were developed with some limitations (Lourenco PB 1995; Brøndum-Nielsen T. 1974; Marti 1991; Gupta AK. 1976; 1981). For the consideration of the material nonlinearity (i.e, concrete), several constitutive models were introduced to reproduce the complex nonlinear behavior of concrete; among the presented models, it is concluded that the concrete damage plasticity model (CDPM) (Jie Li and Ren 2009; Ren, Zeng, and Li 2014; Lee and Fenves 1998; Lubliner et al. 1989) can efficiently simulate and reflect the nonlinear behavior of concrete and converge the results to acceptable accuracy.

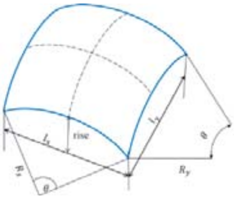
Additionally, for the stochastic analysis, the probability density evolution method (PDEM) was developed based on the principle of preservation(J. Li and Chen 2008; J. Chen and Li 2005), where the SR and the instantaneous PDF of this response can be attained. Additionally, the PDEM combined with the equivalent extreme value event (EEVE) method (J. B. Chen and Li 2007; J. Li, Chen, and Fan 2007)can be adopted to obtain the failure probability based on a proper failure criterion. To the authors' best knowledge, stochastic analysis and reliability quantification for the doubly curved shell structures have not been presented yet.

Thus, this study introduces an intensive investigation of the deterministic and SR of these structures. The developed advanced sandwich model (DASM) is formulated on the equilibrium basis and introduced as an efficient automatic finite element-based design procedure, further employed in this study to obtain steel reinforcement. The multi-axial plasticity damage model and the bilinear kinematic hardening model are adopted to reproduce the complex nonlinear behavior of concrete and steel rebar, respectively. Furthermore, the probabilistic analysis is performed via the PDEM and where both the SR and the instantaneous PDF of the studied roof can be obtained under seismic excitations considering load and material properties uncertainties. Finally, by integrating the PDEM with the EEVE, the structural reliability of the studied shell also could be obtained.

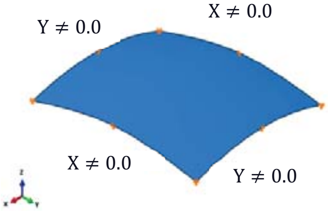
2. Finite element Simulation

2.1. Structural layout of the considered roof

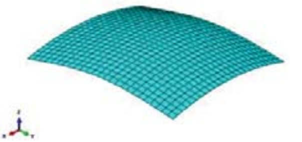
To avoid possible errors in the modeling results, the simulation procedure is verified before starting the simulation and the analysis of the doubly curved roof introduced in the current study. Due to the limited research on curved surfaces both experimentally and theoretically, the analytical solution for the vibration problem of singly and doubly-curved shell surfaces introduced previously by (Ostovari Dailamani 2010) is considered here to validate the simulation procedure. The results showed that the Eigen frequencies as well as mode shapes match well the ones reported in the literature with an average error of 0.5%. The calibrated modeling procedure is applied to more realistic RC doubly curved roofs to cover a plan area of (25m by 25m) with a rise of 3.33m at the maximum curvature, and the included angles are kept equal to 30° in the two perpendicular directions, as shown in Fig.1a. The material properties are adapted to reinforced concrete. The boundary conditions are assumed to be identical to a realistic state where the covering system is supported by walls on the four edges (Leissa AW 2011), as presented in Fig. 1b. According to (ACI Committee 318 2008) for thin concrete shells, the shell thickness is taken as 8 cm. The multi-layer shell element, which is provided by ABAQUS software and commonly utilized to simulate the nonlinear behavior of the slabs and shear walls in the in-plane and out-of-plane directions, is adopted in this study. For the element discretization of the studied roof, the 4-node doubly curved general-purpose shell element with reduced integration (S4R) was considered, and meshing of 1 m is used after mesh convergence analysis; the developed FE model is shown in Fig. 1c.



a) Doubly-curved shell



b) Boundary Conditions



b) Element discretization and meshing

Fig.1. Doubly-curved shell with a plan (25m by 25m)

2.2.Structural design via DASM

Due to the lack in the design codes for the design of such roofs, an efficient finite element-based design procedure (DASM) is employed in this study to obtain steel reinforcement. This design procedure is initially developed by the authors on the basis of equilibrium (Makhloof, Ibrahim, and Ren 2023)] and it is also extended to take into account the transverse shear forces according to (CEN - EN 1992-1-1 2004). In this design procedure, the element is divided into three layers, including the core layer and the top and bottom covers; where the covers are responsible for carrying the membrane forces, bending, and twisting moments through the contribution of the concrete and steel, while the core layer is responsible for the shear forces. The contributions of the concrete and reinforcement resist applied forces and moments. The DASM is formulated and then developed through Python script linked with ABAQUS software to perform an automatic design process and obtain the steel reinforcement for all elements in the roof. The whole formulation of the DASM is omitted here to avoid lengthy. However, the flowchart illustrates the developed Python script is presented in Fig.2. The obtained reinforcement is visualized in Fig.3. It is worthpoiting out that the minimum area of steel according to the design codes is considered for the lower reinforcement ratio. According to the obtained steel reinforcement and the prescribed material models, the nonlinear analysis is performed on the considered shell roof to investigate the nonlinear response under Northridge_01 earthquake. The whole process of investigating this response is depicted in Fig .2.

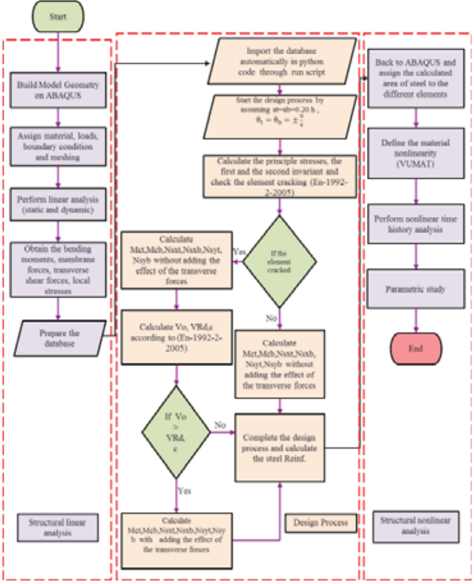
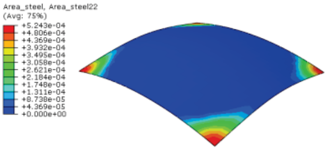
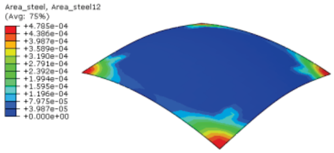


Fig.2 Flow chart for the design process DASM and the nonlinear analysis



a) Reinforcement in circumferential direction



b) Reinforcement in meridian direction

Fig.3. Visualization of the obtained values of steel reinforcement for all elements using the DASM

2.3. Material nonlinearity

In the current simulation procedure, the concrete multi-axial plasticity damage constitutive model is employed to represent the concrete complex nonlinear behavior, whereas the bilinear kinematic hardening model is considered for the steel reinforcement, see Fig.4 for the stress-strain relations of the adopted material models. For the concrete multi-axial plasticity damage constitutive model, in the context of damage mechanics, the modified Hooke's law with damage can be expressed in the multi-dimensional state as follows:

$$\boldsymbol{\sigma} = (\mathbb{I} - \mathbb{D}) : \mathbb{E}_0 : \boldsymbol{\varepsilon} \qquad (1)$$

where $\mathbb{I}$ is the unit tensor, $\mathbb{E}_0$ is the initial modulus of the elasticity tensor, $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ are the stress and strain tensors, respectively, and $\mathbb{D}$ denotes the damage tensor, which indicates the degradation of $\mathbb{E}_0$, and can be represented as follows:

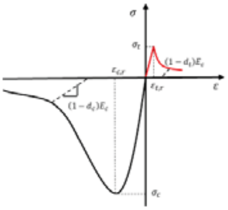
$$\mathbb{D} = \mathrm{D}^{+}\mathbb{P}^{+} + \mathrm{D}^{-}\mathbb{P}^{-} \qquad (2)$$

In this equation, the projective tensors $\mathbb{P}^{+}$ and $\mathbb{P}^{-}$ describe the anisotropies of tensile and compressive degradation, respectively. Also, D^{+} and D^{-} represents the tensile and compressive damage scalars and can be determined as follows:

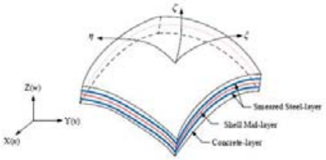
$$\mathrm{d}^{\pm} = \mathrm{g}^{\pm}(Y^{\pm}) \qquad (3)$$

where $\mathrm{g}^{\pm}(\cdot)$ represent the monotonic damage evolution functions, $Y^{\pm}$ denote the damage energy release rates. Based on the consistent damage condition, the concept of energy-equivalent strain, which bridges the gap between the uni-axial and the multi-axial damage evolutions, was proposed (Jie Li and Ren 2009; Jie Li and Wu 2006), and the energy-equivalent strains were presented. Accordingly, the damage evolutionary functions are then formulated and adopted as presented in (Ren, Zeng, and Li 2014; Jie Li and Ren 2009). For the validation and verification of the adopted damage model, it is referred to (Ren, Zeng, and Li 2014; Jie Li and Ren 2009). The nonlinear dynamic response of RC structures is successfully investigated using this model in literature (Ibrahim and Makhloof 2023; Ibrahim, Makhloof, and Ren 2023).

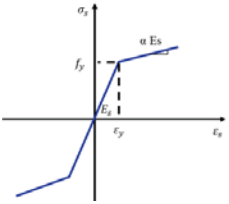
For steel reinforcement, the bilinear kinematic hardening model is considered to reproduce the behavior of the reinforcing steel bars. For the studied roof, reinforced concrete and steel reinforcement with a density of 2400 and 7850 kg/m^3 have been used, respectively. The concrete mechanical properties, including the compressive and tensile strengths, modulus of elasticity, and Poisson's ratio, have been considered as 36 MPa, 3.2 MPa,36000 MPa, 0.2, respectively. Also, for the steel mechanical properties, the modulus of elasticity, yield strength, and Poisson's ratio has been considered to be 200000 MPa, 360 MPa, and 0.3, respectively.



a) Stress-strain relation for concrete



b) Multi-layer shell element



c) Stress-strain relation for steel

Fig.4 Stress-Strain curve for steel and concrete included in the multi-layer shell element

3. Deterministic analysis results of the studied roof

By the nonlinear time history analysis (NLTHA) under intense seismic excitation, the nonlinear dynamic response of the roof under consideration has been investigated. The reinforcement obtained by the DASM was applied to the investigated roof in advance of this analysis, as depicted in Fig. 5.

For this analysis, it is essential to note that, according to previous research, near-fault ground motions can cause more severe structural damage than far-fault ones.

Fig. 5. Assignment of the calculated reinforcement according to the DASM

For this reason, the RC doubly curved shell used in this study is excited by the Northridge_01 event, a near-fault ground motion; for the acceleration time history see Fig.6. The nonlinear dynamic response of the considered roof is investigated in terms of tensile damage of concrete and displacement, as presented in Fig.7. It is worth noting that the concrete tensile damage value provides an indication of the cracking state of the structures within the range of [0 to 1], where Zero represents the elastic state of the structure without any cracking and the unit value, or even when the value is nearly equal to unity, refers to the occurrence of cracks that may be followed by global failure as a result of local failure in a part of the structure.

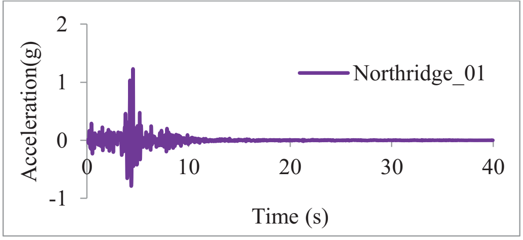
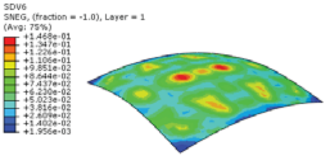
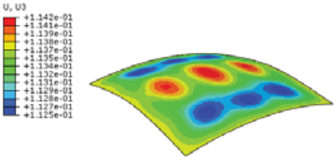


Fig.6. Acceleration time history of the vertical components for the considered excitations

The results demonstrate that the roof shows lower values of tensile damage where the maximum tensile value of concrete was 0.15, as presented in Fig.7a. However, a high displacement response is recorded under the considered seismic excitation, where a maximum value of 11.42 cm is observed, as depicted in Fig.7b. Here, it also should be mentioned that the maximum recorded displacement (11.42 cm) still lower than the maximum allowable deflection u_{max} considered by (ACI Committee 318 2014) where ($u_{max} \leqslant \frac{span}{200}$ = 12.5 cm).



a) Tensile damage contours of the studied roof (SDV6 refers to the tensile damage)



b) Maximum displacement contours

Fig.7. Nonlinear dynamic response of the studied roof under Northridge_01 event

4. Probabilistic analysis via PDEM

4.1 *Consideration of random variables*

The uncertainty in the material properties in addition to the load uncertainty, are considered in the stochastic analysis performed in the current study. The concrete material properties are considered random according to the (Joint Committee on Structural Safety (JCSS). 2001) where a lognormal (LN) distribution is considered for the concrete compressive strength, tensile strength, modulus of elasticity, and the ultimate strain, with mean 1.0 and coefficients of variation (COV) as 0.06, 0.3, 0.15, and 0.15, respectively. The other uncertainty sources, including the steel material properties and load uncertainties, are considered as presented in Table 1.

Table 1. The uncertainty parameters considered for the steel reinforcement and the load

Uncertainty Source	Young's modulus	Yield strength	Acceleration	Dead Load	Live Load
Mean	200000 MPa	350 MPa	1.229 g	3 kN/m^2	1 kN/m^2
COV	0.035	0.085	0.2	0.1	0.4

4.2 The PDEM Fundamentals

In the current study, the probabilistic analysis is performed to investigate the SR of the considered roof and to assess the structural reliability considereing the abovementioned uncertainty sources. In this regard, the PDEM(J. B. Chen and Li 2007; J. Li, Chen, and Fan 2007; Jie Li 2016) is employed, where the PDF surface of the SR and the reliability of the relevant structures can be evaluated. On the basis of the preservation principle of probability, the PDEM was developed, and in case of unique response, Y, the generalized density evolution equation (GDEE) is formulated and expressed as:

$$\frac{\partial p_{Y\Theta}(y,\boldsymbol{\theta},t)}{\partial t}+\dot{Y}(\theta,t)\frac{\partial p_{Y\Theta}(y,\boldsymbol{\theta},t)}{\partial y}=0 \quad (4)$$

In this equation, $y(t)$ represents a vector relating to the response of the structure, such as displacement, acceleration, crack width, or stress at a specific node; as a result, y can be regarded as the state variable, and t denotes the evolution variable. The concerned basic random vector, denoted by the notation Θ (Θ_1 , Θ_2 ,… , Θ_n), influences the parameters related to the structural response, where θ indicates the realized real value that reflects the uncertainty resource, such as loads, material characteristics etc. For a given realization θ of the random vector Θ, $\dot{Y}(\theta,t)$ represents the velocity variable, whereas $\partial p_{Y\Theta}(y,\theta,t)$ represents the joint PDF of $y(t)$ and Θ. The joint PDF can be then presented as follows:

$$p_Y(y,t)=\int_{\Omega_\Theta} p_{Y\Theta}(y,\theta,t)d\theta \quad (5)$$

In the preceding equations, $\boldsymbol{\theta}$ indicates a continuous variable; therefore, for such variables in real-world engineering issues, finite discrete points should be produced. The probability-assigned domain, Ω_Θ, is discretized to accomplish this using an optimal point selection technique, such as the GF discrepancy minimization-based strategy (J. B. Chen and Zhang 2013; J. B. Chen and Chan. 2019; J.-B. Chen, Yang, and Li 2016). The iterative screening-rearrangement method (IS-RAM) effectively creates representative points relying on a minimal GF discrepancy. The chosen point set is represented by the symbols $\theta_k \in \Omega_\Theta$, k = 1, 2,…, N, and $p_\Theta(\theta_k)$ is the relevant assigned probability of θ_k. The initial condition in the numerical analysis using N sample random points can be expressed as follows:

$$p_{Y\Theta}(y,\theta_k,t_0)=\delta(y-y_0)p_\Theta(\theta_k),\ k=1,2,\ldots\ldots,N \quad (6)$$

By integrating (Eq. (6)) and the total variation dimensioning technique (TVD) for the partial equation' solution, it is possible to deduce the solution of $p_{Y\Theta}(y,\theta,t)$ from Eq. (4). As a result, the overall joint PDF can be introduced as follows:

$$p_Y(y,t)=\sum_{k=1}^{N} p_{Y\Theta}(y,\theta_k,t) \quad (7)$$

Additionally, by integrating the PDEM with the EEVE (J. B. Chen and Li 2007; J. Li, Chen, and Fan 2007), the reliability and failure probability of the structure can be obtained based on a defined failure criterion. In this study, since the studied roof showed high displacement response in the deterministic analysis and the displacement can give an indication about the structure state, the maximum displacement (δ_b) is considered as the failure criterion. When δ_b reaches or exceeds the allowable displacement value δ_{alw} according to the design codes (δ_{alw}= Span / 200 = 125 mm), this express the structure failure and vice versa.

4.3 Probabilistic analysis **results** *and structural reliability via PDEM*

Based on the considered random variables, 200 samples were generated, which were significantly reduced using the GF discrepancy minimization-based method. Then, the deterministic analyses for the generated samples are performed for each sample to get the state and evolution variables. Subsequently, the stochastic analysis is conducted according to the PDEM fundamentals, and the SR is explored. As observed in Fig.7b, the studied roof showed the relatively highest displacement at the node of the maximum curvature. Therefore, in the stochastic analysis, this node is selected as the target node. Fig.8 illustrates the displacement-time curves for the generated samples

Subsequently, the SR is continued to be explored via the PDEM. The PDF surface is presented in Fig.9, where the displacement and time are considered as the state and evolution variables, respectively. Additionally, the PDFs at a special time of interest are depicted in Fig.10. It can be seen from the PDF curves at certain points of interest that the normal distribution curve's shape includes wide displacement ranges. Also, at a higher amplitude of the considered seismic wave, the structure suffers a higher displacement response which means that the structure could enter the nonlinear stage; at these stages, a wide range of displacement can be observed from the PDF curves. For instance, at time 5.3 s, a wide range for the displacement from 20 mm to 120 mm, also at 5.8 s, the displacement ranges from 60 mm to 160 mm, which proves and demonstrates the uncertainty effect on the performance of the structure which increases with the increasing of structural nonlinearity. In the stochastic analysis, a maximum displacement of 155 cm is recorded for some samples, which is relatively 1.8 times the maximum displacement recorded in the deterministic analysis, indicating that the deterministic analysis cannot reproduce the real behavior of the structure or quantify its reliability.

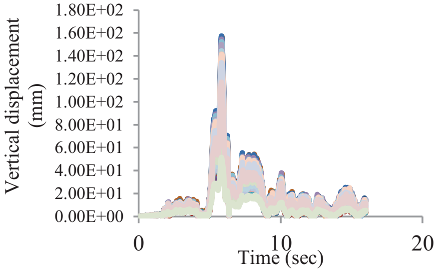


Fig.8. The SR of the generated samples

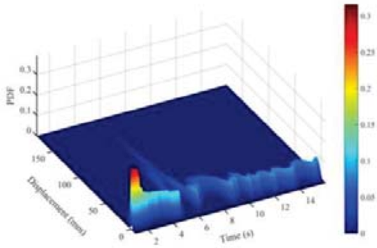


Fig.9. The PDF surface

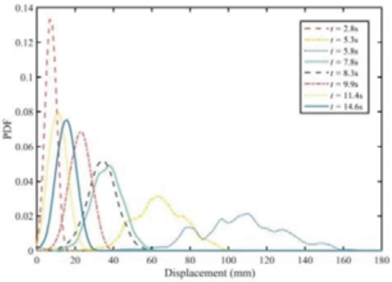


Fig.10. PDFs of special times of interest

According to the stochastic analysis results and the predefined failure criterion, the failure of the studied roof occurs when the displacement exceeds the allowable displacement of 125 mm. Based on the PDEM combined with the EEVE and the adopted limit state, the reliability curve can be attained, as depicted in Fig.11, where at the considered limit state (displacement equal 125mm), the structure reliability is 70.78 % which means that the studied roof exhibits nearly 29.22 % failure probability . Eventually, these results indicate that when considering material and load uncertainties, the structure exhibits various responses under the same seismic event, which cannot be revealed in the deterministic analysis.

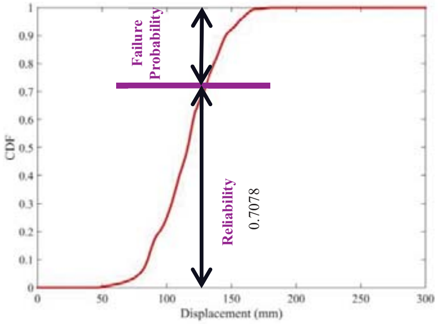


Fig.11. Reliability curve for the studied roof

5. Conclusion

This study introduces an efficient framework that incorporates the developed advanced sandwich model (DASM), the multi-axial concrete damage plasticity model (CDPM), and the probability density evolution method (PDEM) to investigate the stochastic response and assess the structural reliability.The developed advanced sandwich model (DASM) can be efficiently used to design the shell structures where the design process can be completed automatically in a few seconds on a standard computer with accurate results. The results of this study demonstrated the significant influence of considering the uncertainties in loads and materials properties on the performance of RC doubly curved shells showing high resistance in some cases and undesired consequences in others. In the stochastic analysis, a maximum displacement of 155 cm is recorded for some cases, which is relatively 1.8 times the maximum displacement recorded in the deterministic analysis. This means that deterministic analysis discards these uncertainties and cannot reproduce the real behavior of the structure, the structural reliability can not be quantified via this analysis. Unlike the results of the deterministic analysis, where the maximum displacement is less than the maximum allowable displacement, the buckling failure would not occur. The stochastic analysis revealed that buckling failure could occur and the probability of failure is approximately 29.22 %, respectively, indicating the need for considering the different uncertainty sources in the analysis and design of these structures.

It should be noted here that one of the limitations of this study that some material properties are considered uncorrelated random variables, however they are likely correlated which will be considered in the future research.

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