Proceedings of the 33rd European Safety and Reliability Conference (ESREL 2023)
Edited by Mário P. Brito, Terje Aven, Piero Baraldi, Marko Čepin and Enrico Zio

doi: 10.3850/978-981-18-8071-1_P109-cd

An active learning reliability analysis framework based on multi-fidelity surrogate model

Ning Lu

School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, China.
Center for System Reliability and Safety, University of Electronic Science and Technology of China, China.
E-mail: ninglu@std.uestc.edu.cn

Yan-Feng Li

School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, China.
Center for System Reliability and Safety, University of Electronic Science and Technology of China, China.
E-mail: yanfengli@uestc.edu.cn

Song Bai

School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, China.
Center for System Reliability and Safety, University of Electronic Science and Technology of China, China.
E-mail: baisong@std.uestc.edu.cn

Tudi Huang

School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, China.
Center for System Reliability and Safety, University of Electronic Science and Technology of China, China.
E-mail: huangtudi@std.uestc.edu.cn

Hong-Zhong Huang

School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, China.
Center for System Reliability and Safety, University of Electronic Science and Technology of China, China.
E-mail: hzhuang@uestc.edu.cn

On the one hand, the limit state function (LSF) can be used to define structural reliability, while complex structures frequently correspond to LSFs with high computational costs, necessitating numerous calls to LSFs for the reliability analysis of such structures. On the other hand, detailed paradigms and coarse paradigms are generally considered high-fidelity (HF) models with low model uncertainty and low-fidelity (LF) models with low computational cost, respectively. To effectively address both of these common challenges, this paper proposes an active learning multi-fidelity surrogate modeling framework for structural reliability analysis (SRA). The multi-fidelity (MF) surrogate model has received widespread attention in the performance evaluation of complex structures by fusing models with different accuracies to reduce the computational demand and effectively balance the prediction performance and modeling cost of the surrogate model. In three numerical examples and one engineering example in different dimensions, two multi-fidelity surrogate models with four learning functions are tested and compared with the corresponding single-fidelity (SF) models. All the results demonstrate that the MF model based on this framework is more efficient than the SF model at reducing computational costs without compromising accuracy.

Keywords: reliability analysis, active learning, multi-fidelity, surrogate model, aero engine gear.

1. Introduction

Limit state function (LSF) can be used to define structural reliability, while complex structures frequently correspond to LSFs with high computational costs, necessitating numerous calls to LSFs for the reliability analysis of such structures. In this context, the surrogate model can approximate true LSF effectively to model complex LSF at lower computational costs, and active learning can be further introduced to achieve modeling accuracy with fewer calls to LSF by adaptively selecting learning samples through the machine learning algorithm. Both can be used to respond to the challenge of balancing accuracy and efficiency in structural reliability analysis (SRA) and are rapidly becoming essential methods for the efficient evaluation of structural reliability. Accordingly, active learning surrogate model-based SRA is used in a wide range of fields, involving aeronautics and aerospace, automotive industry, civil engineering, critical infrastructures, land transportation, maritime and offshore technology, nuclear industry, railway industry, water transportation, etc.

Nevertheless, the same analysis can correspond to multiple models with different paradigms (e.g., experiment, theory, simulation, big data), and different paradigms or even the same paradigm usually correspond to different fidelities. Detailed paradigms and coarse paradigms are generally considered high-fidelity (HF) models with low model uncertainty and low-fidelity (LF) models with low computational cost, respectively. Therefore, a similar challenge also exists in surrogate modeling, i.e., obtaining the quantity of interest (QOI) at an acceptable cost level by selecting models with appropriate fidelity. As an effective method to deal with this problem, the multi-fidelity (MF) surrogate model has received widespread attention in the performance evaluation of complex structures by fusing models with different accuracies to reduce the computational demand and effectively balance the prediction performance and modeling cost of the surrogate model.

To effectively address both of these common challenges, this paper proposes an active learning multi-fidelity surrogate modeling framework for SRA: firstly, a multi-fidelity Kriging (MFK) or a multi-fidelity Gaussian process (MFGP) is modeled based on the theory of surrogate model and multi-fidelity; secondly, learning functions EIF (expected improvement function), EFF (expected feasibility function), U or H and its stopping criterion are applied to implement active learning; finally, Monte Carlo simulation (MCS) is used to implement reliability evaluation. This framework incorporates different fidelity information in the form of online data-driven to accomplish the trade-off between high prediction accuracy and low computational cost by combining HF and LF models.

2. Proposed Framework

An active learning multi-fidelity surrogate framework proposed in this paper is implemented as follows:

Step 1. Generate N_{MCS} samples $\boldsymbol{x}_{\mathrm{MCS}}$ with QMC (quasi-Monte Carlo), forming a sample pool S_{MCS}.

Step 2. Draw N_{HF} HF samples $\boldsymbol{x}_{\mathrm{HF}}$ and N_{LF} LF samples $\boldsymbol{x}_{\mathrm{LF}}$ in S_{MCS} as initial training samples following nested sampling i.e., $\boldsymbol{x}_{\mathrm{HF}} \subset \boldsymbol{x}_{\mathrm{LF}}$ (Le Gratiet, 2013; Jin et al., 2005; Park et al., 2017), and $N_{\mathrm{HF}}/N_{\mathrm{LF}} = 1/3$ (Lv, 2020), and calculate their responses $g_{\mathrm{HF}}(\boldsymbol{x}_{\mathrm{HF}})$, $g_{\mathrm{LF}}(\boldsymbol{x}_{\mathrm{LF}})$. Form the initial training set T.

Step 3. Use the training set T to construct the MF surrogate model g_{MF} based on the mixed scale.

Step 4. Calculate $g_{\mathrm{MF}}(\boldsymbol{x}_{\mathrm{MCS}})$ based on g_{MF} and calculate the learning function.

Step 5. Determine the stopping condition of the learning function: if it is satisfied, go to Step 7; otherwise, go to Step 6.

Step 6. Determine an additional sample $\boldsymbol{x}_{\mathrm{HF}}$ based on the learning function, and further determine the corresponding three $\boldsymbol{x}_{\mathrm{LF}}$ and the responses of these four samples to update the training set T, and return to Step 3.

Step 7: Calculate the coefficient of variation of the failure probability estimation: if it is less than 5%, output the estimation and the process ends;

otherwise, expand S_{MCS} with QMC and return to Step 4.

3. Academic Validation

Denote the HF/LF LSF as $g_{\mathrm{HF}}(\boldsymbol{X})$ and $g_{\mathrm{LF}}(\boldsymbol{X})$, respectively, *cost* as 1.0 × (number of initial HF samples + number of added HF samples) + 0.1 × (number of initial LF samples + number of added LF samples), and *error* as the error relative to MCS.

3.1. *One-dimensional example*

To observe the effect of the model at the failure boundary, the HF model (Sasena et al., 2002; Huang et al., 2006; Hu, 2019) and LF model (Huang et al., 2006; Hu, 2019) are overall translated to obtain a modified 1D HF/LF LSF, as shown in Eq. (1), Table 1, and Fig. 1. The results are shown in Table 2 and Fig. 2.

For this 1D example: H converges early, leading to the largest error; the U is generally costly and over-convergence is more obvious in MFGP+U; due to the extremely low dimensionality, a very small number of HF samples can obtain high surrogate accuracy, and LF samples tend to adversely affect MF surrogates, such as MFGP+EIF; MF surrogates do not show the advantage.

$$\begin{cases} g_{\mathrm{HF}}(\boldsymbol{X}) = -\sin(X) - e^{\frac{X}{100}} + 2 \\ g_{\mathrm{LF}}(\boldsymbol{X}) = -\sin(X) - e^{\frac{X}{100}} + 0.03(X-3)^2 - 8 \end{cases} \quad (1)$$

Table 1. Example 1, input variables.

Variable	Distribution	minimum	maximum
X	Uniform	0	10

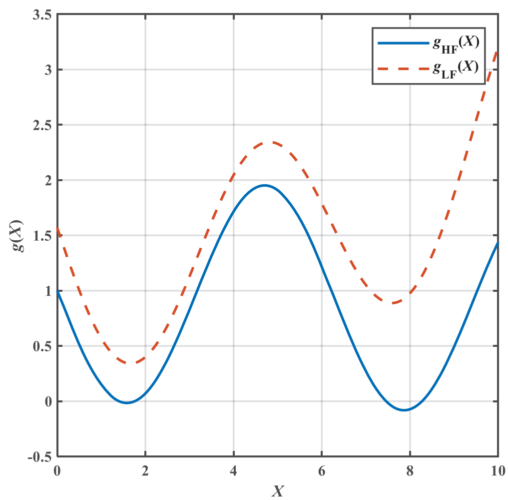

Fig. 1. Example 1, LSF.

Table 2. Example 1, reliability analysis results.

Method	Learning function	*cost*	P_{f}	Coefficient of variation	*error* (%)
MCS	N/A	10^7	0.1178500	0.0008	N/A
Kriging	EIF	5+5	0.1170000	0.0433	0.7213
	EFF	5+7	0.1172500	0.0434	0.5091
	U	5+7	0.1172500	0.0434	0.5091
	H	5+4	0.0856250	0.0434	27.3441
GP	EIF	5+5	0.1172500	0.0434	0.5091
	EFF	5+6	0.1172500	0.0434	0.5091
	U	5+6	0.1172500	0.0434	0.5091
	H	5+4	0.1162500	0.0308	1.3577
MFK	EIF	5+6+0.1×(15+18)	0.1175000	0.0433	0.2970
	EFF	5+6+0.1×(15+18)	0.1175000	0.0433	0.2970
	U	5+7+0.1×(15+21)	0.1175000	0.0433	0.2970
	H	5+5+0.1×(15+15)	0.0356250	0.0411	69.7709
MFGP	EIF	5+3+0.1×(15+9)	0.0890000	0.0358	24.4803
	EFF	5+5+0.1×(15+15)	0.1178125	0.0216	0.0318
	U	5+9+0.1×(15+27)	0.1171250	0.0153	0.6152
	H	5+2+0.1×(15+6)	0.0513750	0.0480	56.4064

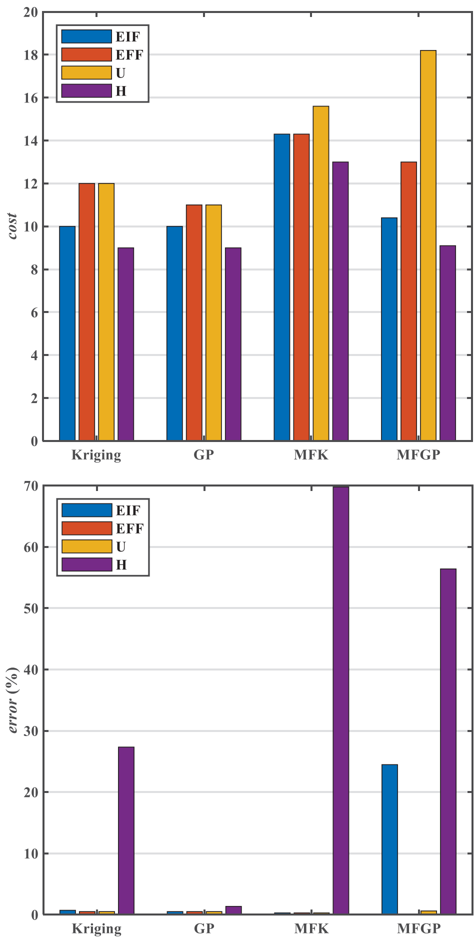

Fig. 2. Example 1, cost and error.

3.2. *Two-dimensional example*

Similarly, the translation of the model (Xuan, 2020) is performed to obtain a modified 2D HF/LF LSF, as shown in Eq. (2), Table 3, and Fig. 3. The results are shown in Table 4 and Fig. 4.

$$\begin{cases} g_{\mathrm{HF}}(\boldsymbol{X}) \\ = X_1^2 + X_2^2 - 10\cos 2\pi X_2 + 15.5 \\ g_{\mathrm{LF}}(\boldsymbol{X}) \\ = 0.5\left(X_1^2 + X_2^2 - 10\cos 2\pi X_1 - 10\cos 2\pi X_2\right) - \\ 3(X_1 + X_2) + 6 \end{cases} \tag{2}$$

Table 3. Example 2, input variables.

Variable	Distribution	Mean	Standard deviation
X_1	Normal	1	0.5
X_2	Normal	0	0.5

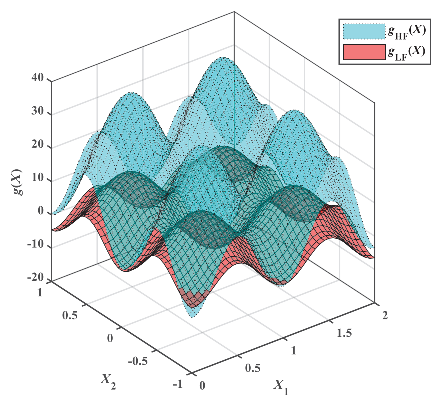

Fig. 3. Example 2, LSF.

Table 4. Example 2, reliability analysis results.

Method	Learning function	*cost*	P_f	Coefficient of variation	*error* (%)
MCS	N/A	10^7	0.0525278	0.0013	N/A
Kriging	EIF	12+57	0.0491250	0.0492	6.4781
	EFF	12+95	0.0502500	0.0486	4.3364
	U	12+82	0.0486062	0.0486	7.4658
	H	12+80	0.0501250	0.0487	4.5743
GP	EIF	12+91	0.0505000	0.0485	3.8604
	EFF	12+147	0.0496250	0.0489	5.5262
	U	13+94	0.0501250	0.0487	4.5743
	H	12+139	0.0496250	0.0489	5.5262
MFK	EIF	12+73+0.1×(36+219)	0.0503438	0.0172	4.1579
	EFF	12+96+0.1×(36+288)	0.0501250	0.0487	4.5743
	U	12+72+0.1×(36+216)	0.0502500	0.0486	4.3364
	H	12+140+0.1×(36+420)	0.0519297	0.0119	1.1387
MFGP	EIF	12+30+0.1×(36+90)	0.0502500	0.0486	4.3364
	EFF	12+45+0.1×(36+135)	0.0535000	0.0470	1.8508
	U	12+42+0.1×(36+126)	0.0506250	0.0484	3.6225
	H	12+50+0.1×(36+150)	0.0533571	0.0356	1.5789

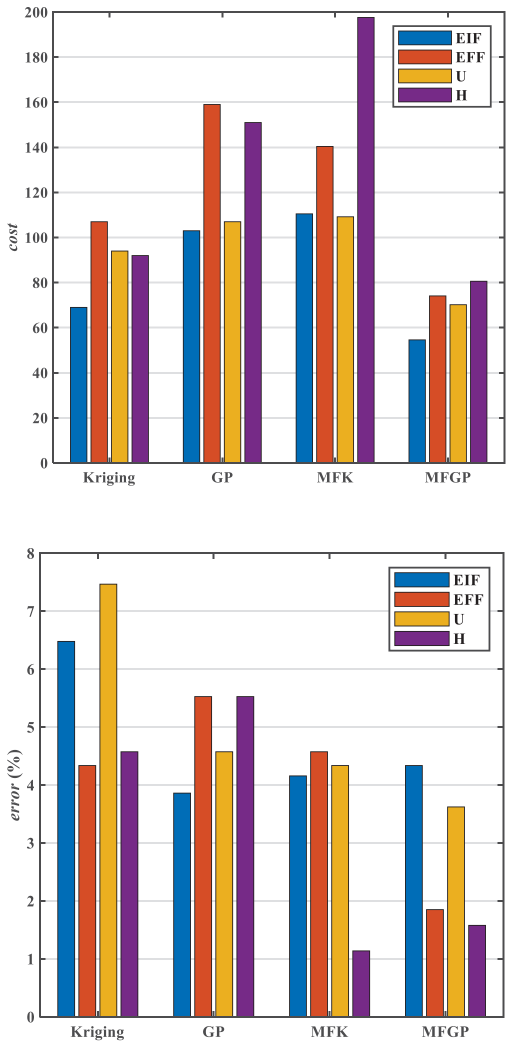

Fig. 4. Example 2, cost and error.

For this 2D example: the learning function with cost advantage tends to have unsatisfactory error, thus it is difficult to directly conclude the superiority of the learning function; H is towards over-convergence; MFGP has more cost advantage and less error with the same learning function; EFF works better when combined with MFGP; as the dimension increases to two, the MF model starts to show its advantages.

3.3. *Six-dimensional example*

A 6D HF/LF LSF (Xuan, 2020) is shown in Eq. (3) and Table 5. One hundred samples were randomly sampled and their responses are shown in Fig. 5. The results are shown in Table 6 and Fig. 6.

$$
\begin{cases}
g_{\mathrm{HF}}(\boldsymbol{X}) \\
= 25(X_1-2)^2+(X_2-2)^2+(X_3-1)^2+ \\
(X_4-4)^2+(X_5-1)^2+(X_6-4)^2-5 \\
g_{\mathrm{LF}}(\boldsymbol{X}) \\
= 0.3\begin{pmatrix} 25(X_1-2)^2+(X_2-2)^2+(X_3-1)^2+ \\ (X_4-4)^2+(X_5-1)^2+(X_6-4)^2 \end{pmatrix}- \\
2(X_2-1)^2-2(X_6-4)^2
\end{cases}
\tag{3}
$$

Table 5. Input variables.

Variable	Distribution	Mean	Standard deviation
X_1	Normal	1	2
X_2	Normal	0	1.5
X_3	Normal	1.5	1
X_4	Normal	2.5	2
X_5	Normal	2	2
X_6	Normal	3	2

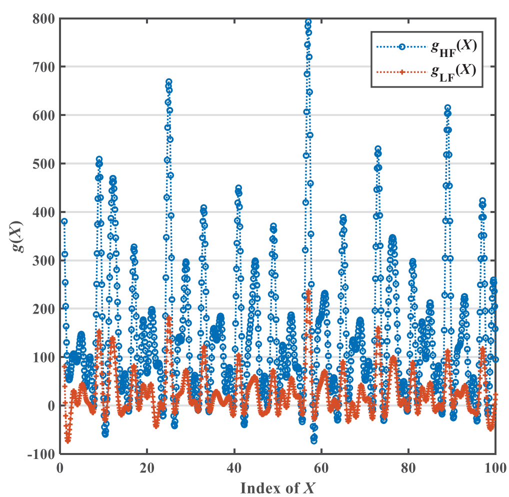

Fig. 5. Example 3, response of samples.

Table 6. Example 3, reliability analysis results.

Method	Learning function	*cost*	P_f	Coefficient of variation	*error* (%)
MCS	N/A	10^7	0.0030590	0.0057	N/A
Kriging	EIF	12+52	0.0029453	0.0364	3.7165
	EFF	12+89	0.0029414	0.0364	3.8442
	U	12+87	0.0029375	0.0364	3.9719
	H	12+55	0.0028945	0.0367	5.3766
GP	EIF	12+27	0.0029375	0.0364	3.9719
	EFF	12+37	0.0029375	0.0364	3.9719
	U	12+50	0.0029414	0.0364	3.8442
	H	12+31	0.0029453	0.0364	3.7165
MFK	EIF	12+32+0.1×(36+96)	0.0029492	0.0363	3.5888
	EFF	12+91+0.1×(36+273)	0.0029180	0.0365	4.6104
	U	12+74+0.1×(36+222)	0.0029180	0.0365	4.6104
	H	12+69+0.1×(36+207)	0.0029219	0.0365	4.4827
MFGP	EIF	12+28+0.1×(36+54)	0.0029883	0.0361	2.3118
	EFF	12+98+0.1×(36+294)	0.0029219	0.0365	4.4827
	U	12+90+0.1×(36+270)	0.0029297	0.0365	4.2273
	H	12+35+0.1×(36+105)	0.0029688	0.0362	2.9503

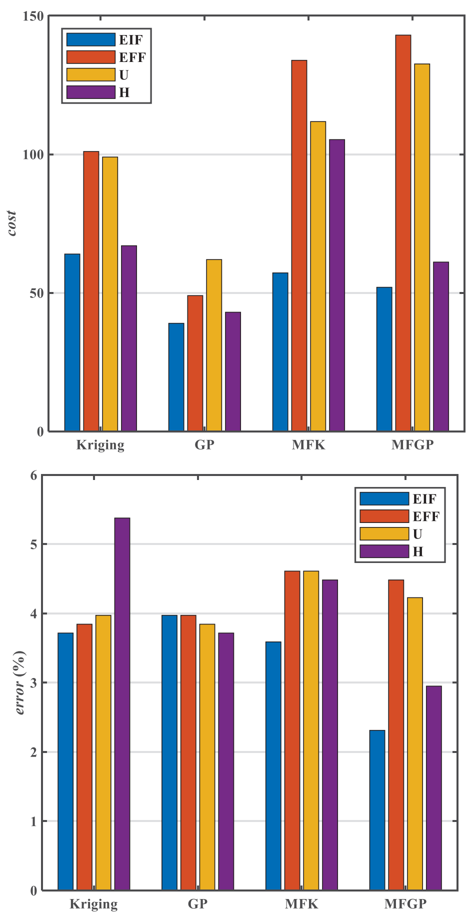

Fig. 6. Example 3, cost and error.

For this 6D example: EIF and H have lower costs and smaller errors, especially the combination with MFGP has the smallest error and the cost is not much different from other methods; GP has the lowest cost and acceptable error; with the further increase of dimensionality, the combined advantage of MF model is more obvious, especially MFGP+EIF/H.

4. Engineering Application

Consider the tooth flank contact stress of a pinion from a gear pair in the fatigue contact test for the aero engine. According to international standard (ISO, 2019):

$$\begin{aligned}\sigma_{\mathrm{H1\,HF}} &= Z_{\mathrm{B}}\sigma_{\mathrm{H0}}\sqrt{K_{\mathrm{A}}K_{\gamma}K_{\mathrm{v}}K_{\mathrm{H}\beta}K_{\mathrm{H}\alpha}} \\ &= Z_{\mathrm{B}}Z_{\mathrm{H}}Z_{\mathrm{E}}Z_{\varepsilon}Z_{\beta}\sqrt{\frac{F_{\mathrm{t}}}{d_1 b_2}\frac{u+1}{u}K_{\mathrm{A}}K_{\gamma}K_{\mathrm{v}}K_{\mathrm{H}\beta}K_{\mathrm{H}\alpha}}\end{aligned} \tag{4}$$

According to Hertzian contact mechanics (Hertz, 1882):

$$\begin{aligned}\sigma_{\mathrm{H1\,LF}} &= \sqrt{\frac{F_{\mathrm{n}}\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)}{\pi L\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}} \\ &= Z_{\mathrm{H}}Z_{\mathrm{E}}Z_{\varepsilon}\sqrt{\frac{F_{\mathrm{t}}}{d_1 b_2}\frac{u+1}{u}}\end{aligned} \tag{5}$$

Therefore, the HF/LF LSF for tooth flank contact fatigue failure is

$$\begin{cases} g_{\mathrm{HF}}\left(\boldsymbol{X}\right)=\sigma_{\mathrm{HP}}-\sigma_{\mathrm{H1\,HF}} \\ g_{\mathrm{LF}}\left(\boldsymbol{X}\right)=\sigma_{\mathrm{HP}}-\sigma_{\mathrm{H1\,LF}} \end{cases} \tag{6}$$

where σ_{HP} is the permissible contact stress, which is calculated according to Eq. (7).

$$\sigma_{\mathrm{HP}}=\frac{\sigma_{\mathrm{H\,lim}}Z_{\mathrm{NT}}}{S_{\mathrm{H\,min}}}Z_{\mathrm{L}}Z_{\mathrm{v}}Z_{\mathrm{R}}Z_{\mathrm{W}}Z_{\mathrm{X}} \tag{7}$$

The input vector $\boldsymbol{X}=\left[m_{\mathrm{n}},b_2,E,\nu,T_1\right]^{\mathrm{T}}$ is shown in Table 7, and the response of one hundred random samples is shown in Fig. 7.

Table 7. Example 4, input variables.

Symbol (unit)	Mean	Coefficient of variation	Description
m_{n} (mm)	5	0.001	Normal module
b_2 (mm)	24	0.01	Face width of wheel
E (Pa)	2.07×10^{11}	0.01	Modulus of elasticity
ν	0.298	0.01	Poisson's ratio
T_1 (N·m)	145	0.1	Torque of pinion

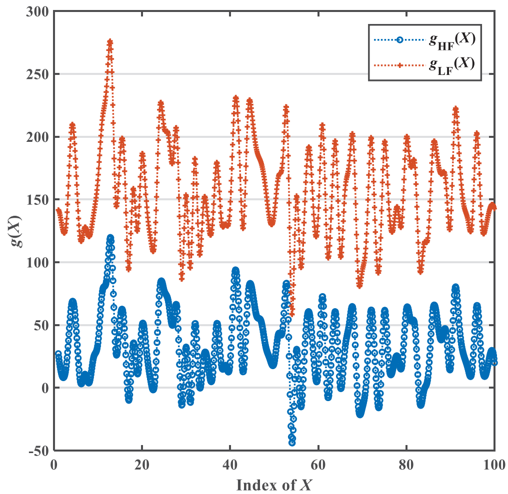

Fig. 7. Example 4, response of samples.

For this engineering example: EIF and H are more advantageous in cost, but the comprehensive effect of U is more stable; the error of MFGP+EIF/H is extremely low, and the difference in cost among other methods is not large; in general, GP and MFK are more compatible with different learning functions.

Table 8. Example 4, reliability analysis results.

Method	Learning function	*cost*	P_{f}	Coefficient of variation	*error* (%)
MCS	N/A	10^7	0.0608430	0.0012	N/A
Kriging	EIF	12+4	0.0638750	0.0428	4.9833
	EFF	12+15	0.0610000	0.0439	0.2580
	U	12+8	0.0610000	0.0439	0.2580
	H	12+2	0.0621250	0.0434	2.1071
GP	EIF	12+4	0.0611250	0.0438	0.4635
	EFF	12+4	0.0610000	0.0439	0.2580
	U	12+5	0.0610000	0.0439	0.2580
	H	12+4	0.0610000	0.0439	0.2580
MFK	EIF	12+4+0.1×(36+12)	0.0610000	0.0439	0.2580
	EFF	12+14+0.1×(36+42)	0.0607500	0.0440	0.1529
	U	12+8+0.1×(36+18)	0.0610000	0.0439	0.2580
	H	12+3+0.1×(36+9)	0.0611250	0.0438	0.4635
MFGP	EIF	12+6+0.1×(36+18)	0.0608750	0.0439	0.0526
	EFF	12+65+0.1×(36+195)	0.0583750	0.0449	4.0563
	U	12+25+0.1×(36+75)	0.0605000	0.0441	0.5637
	H	12+2+0.1×(36+6)	0.0607500	0.0440	0.1529

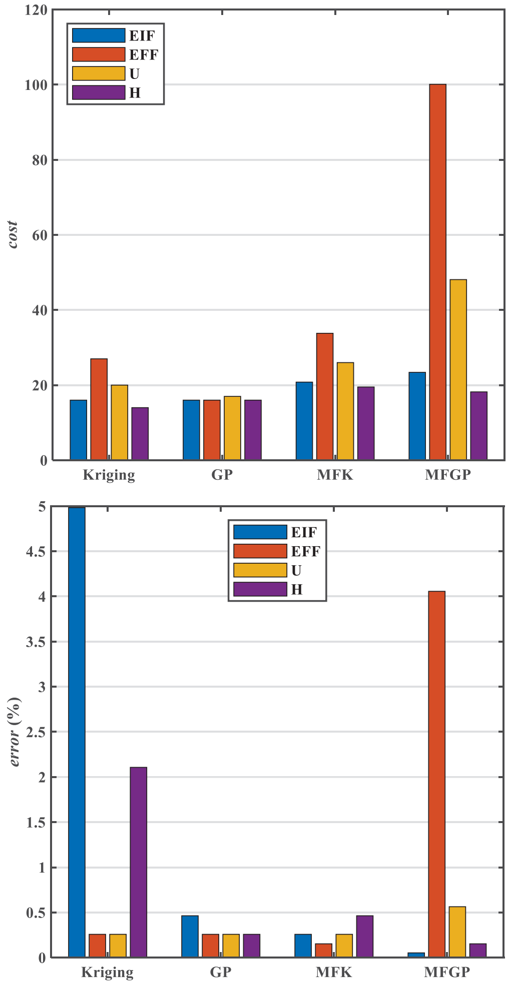

Fig. 8. Example 4, cost and error.

5. Conclusion

In three numerical examples in different dimensions, two multi-fidelity surrogate models with four learning functions are tested and compared with the corresponding single-fidelity (SF) models, which validate the effectiveness of the proposed framework. For the engineering example of aero engine gear in contact fatigue test, the HF LSF and the LF LSF are constructed based on the standard formulation and the simplified formulation respectively. The proposed framework is used for its SRA and the superiority is proved. All the results demonstrate that the MF model based on this framework is more efficient than the SF model at reducing computational costs without compromising accuracy.

Acknowledgement

This research is supported by the National Science and Technology Major Project under the grant number J2019-IV-0001-0068.

References

Hertz, H. (1882). Über die Berührung Fester Elastischer Körper (on the contact of elastic solids). J Reine und Angewandte Mathematik 92, 156–171.

Hu, J. (2019). Improved Hierarchical Kriging Model and its Application in Simulation Model Validation. PhD dissertation, Huazhong University of Science and Technology.

Huang, D., Allen, T.T., Notz, W.I., and Miller, R.A. (2006). Sequential kriging optimization using multiple-fidelity evaluations. Structural and Multidisciplinary Optimization 32 (5), 369–382.

ISO. 2019. 6336-2. Calculation of load capacity of spur and helical gears — Part 2: Calculation of surface durability (pitting) Geneva, Switzerland: International Organization for Standardization.

Jin, R., Chen, W., and Sudjianto, A. (2005). An efficient algorithm for constructing optimal design of computer experiments. Journal of Statistical Planning and Inference 134 (1), 268–287.

Le Gratiet, L. (2013). Multi-fidelity Gaussian process regression for computer experiments. PhD dissertation, Université Paris-Diderot-Paris VII.

Lv, L. (2020). On Advanced Surrogate Models and Application. PhD dissertation, Dalian University of Technology.

Park, C., Haftka, R.T., and Kim, N.H. (2017). Remarks on multi-fidelity surrogates. Structural and Multidisciplinary Optimization 55 (3), 1029–1050.

Sasena, M.J., Papalambros, P., and Goovaerts, P. (2002). Exploration of Metamodeling Sampling Criteria for Constrained Global Optimization. Engineering Optimization 34 (3), 263–278.

Xuan, Z. (2020). Research on Reliability Analysis Method Based on Multi-fidelity Approximation Model. MA thesis, Huazhong University of Science and Technology.