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Comparison of a Normal and Logistic Probability Distribution for the Determination of the Impact Resistance of Polycarbonate Vision Panels

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International standards for the safety of machinery define requirements for the design of safeguards in machine tools. An essential requirement of the safeguard consists of retaining ejected workpiece or tool fragments in case of an accident. Appropriate protective performance of the guard is demonstrated by means of an impact test carried out against a standardized projectile. The impact resistance (IR) is generally used as quantitative measure of an appropriate protective performance. It is defined as maximum kinetic projectile energy a safeguard is able to withstand. The standard procedure for determining the IR is the so-called bisection method, which involves narrowing a wide interval through a series of impact tests. However, this approach is associated with considerable uncertainty since it depends solely on the last two impact tests. In the present study, an alternative approach based on a probabilistic description of failed impact tests is proposed. A normal and a logistic distribution are compared in terms of their suitability for modeling the probability of failed impact tests. Both distributions are well suited, albeit the normal distribution requires considerable data preparation, which also affects its results. In contrast, the logistic distribution does not require any data preparation, providing an advantage over the normal distribution. This new approach can reduce the uncertainty associated with the determination of the IR providing more accurate and reliable results.

Keywords: Safety of machinery, polycarbonate, impact resistance, ballistic limit velocity, probability distribution

1. Introduction

The Directive 2006/42/CE (2006) – also known as Machinery Directive (MD) – of the EUROPEAN UNION specifies fundamental safety requirements for the design and construction of machine tools and in particular, safeguards. This includes, among other things, the requirement that no part, e. i. workpiece or tool fragments, "must be ejected" (Directive 2006). In order to meet these criteria international standards such as ISO 14120 (2015) define specific test procedures to verify an adequate level of protection. In those test procedures a safeguard is subjected to a high energy impact by a standardized projectile. The damage pattern of the safeguard is subsequently used to assess the test result. A test is regarded as passed if it leads only to elastic-plastic deformations with incipient cracks. As soon as the deformation yields a continuous crack visible on both sides of the safeguard, the test is considered failed. As a measure for the protective performance of safeguards standards use the impact resistance (IR) Y (ISO 14120 2015). It is defined as maximum kinetic projectile energy E_{pr} a safeguard is able to withstand and is generally determined applying the so-called bisection method. In this method, the IR Y is estimated by an initially wide interval, which is subsequently narrowed by a series of further impact tests. Although simple in principle, it yields only an interval for the IR Y, which ultimately depends entirely on the two nearest impact test results and thus is associated with considerable uncertainty (LANDI 2022C).

An alternative approach for the determination of the IR Y was presented by UHLMANN ET AL. (2022), who proposed a statistical evaluation procedure. The statistical method allows a probabilistic description of the IR Y by means of cumulative distribution function (CDF) of a normal distribution. Hence, instead of determining a fixed interval, the IR Y is described as probability P of passing an impact test.

UHLMANN ET AL. (2022) successfully applied the novel statistical approach to several impact test series on polycarbonate (PC) sheets. Although there has been a theoretical discussion on employing a normal distribution, a thorough analysis of its suitability is yet to performed.

The aim of this paper is to conduct such an analysis and thus provide a sound fundament for the further application of the statistical approach. For this purpose, a large dataset containing a total number of $n_{PC} = 104$ impact tests on PC-sheets is analyzed. The results are subsequently mapped with a normal distribution. In addition, the suitability of a logistic distribution for modelling the IR Y is investigated. A comparison between the distributions indicate that the IR Y can be expressed by both – a normal and a logistic distribution whereas both distributions exhibit a coefficient of determination $R^2 > 0.86$.

2. Experimental data

The impact test data for the present study was originally obtained by UHLMANN ET AL. (2019), who analyzed the deteriorating effect of cooling lubricants (CL) on PC. The study included bent strip as well as tensile and impact tests. For the impact tests a total number of $n_{PC} = 104$ PC-sheets were exposed to CL. Square PC-sheets with a width of $w_{PC} = 300$ mm and a thickness of $t_{PC} = 12$ mm were subjected to impact tests according to ISO 23125 (2015). The impact tests were carried out in a test facility of the INSTITUTE FOR MACHINE TOOLS AND FACTORY MANAGEMENT (IWF) of the TU BERLIN, see Fig. 1. A detailed description of experimental setup of this investigation can be found in UHLMANN ET AL. (2019).

The results of the bent strip and tensile tests showed a significant degradation of the mechanical properties due to CL-exposure. The impact test results, however, were less conclusive. Although a maximum decrease in IR Y of $\Delta Y = 10$ % was observed, it was also apparent, that the results were subjected to pronounced fluctuations, such that it remained uncertain whether the observed decrease in IR Y was indeed CL-related. Applying the novel statistical approach to the impact test results, UHLMANN ET AL. (2022) were able to conduct a hypothesis test and address this question. No evidence of a CL-related decrease in IR Y was found by this analysis.

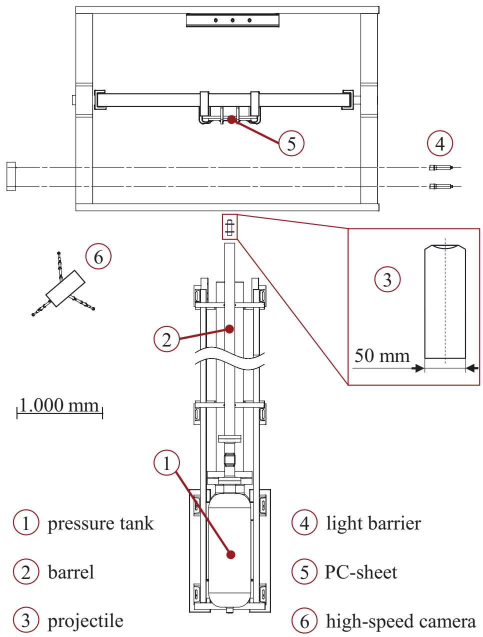

Fig. 1. Impact test facility at the IWF

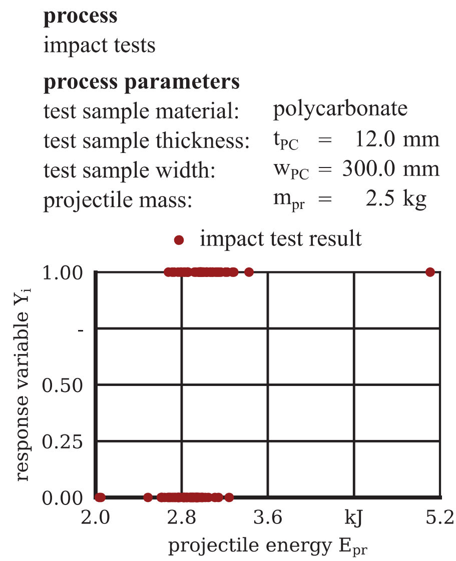

Fig. 2. Impact test results

The observed decrease in IR Y of $\Delta Y = 10\,\%$ was thus attributed to statistical scatter, caused by the complex test conditions associated with impact tests and a typically small number n_{PC} of test samples for the individual impact test series. Since no CL-related aging of PC was found in the original study, the results allow for an in-depth examination of the statistical approach and the underlying distribution. In the absence of aging, from a statistical point of view the entire dataset of impact tests can be treated as a single sample. Hence, the total number of impact tests of $n_{PC} = 104$ from this investigation can now be employed in the present study, which significantly contributes to the validity of this analysis.

3. Data preparation

Fig. 2 shows complete dataset of the impact test results. When presenting qualitative data such as impact test results, it is common to use a binary response variable Y_i that yields either a "success" or a "failure (MONTGOMERY AND RUNGER 2014).

In the present case a response variable of $Y_i = 0$ corresponds to a failed impact test according to ISO 23125 (2015) and vice versa. Since a normal distribution is poorly suited for fitting to binary data, the impact test results must be processed in advance. Therefore, the binary data is transformed to quantitative data by classifying the impact test results in terms of projectile energy E_{pr} into ranges and calculating the probability P for a failed test (UHLMANN ET AL. 2022). It is generally intended to obtain as many ranges as possible, as this provides a larger number of support points and thus enhances the goodness of the normal distribution fit. Conversely, it is beneficial to have as many impact test results in a single range, as this maximizes the accuracy of the calculated probability P. As additional constraint, it is necessary for each range to contain a sufficient number of impact tests n_{PC}. Fig. 3 shows the entire dataset classified into 16 ranges and the number of impact tests n_{PC} for each range. Two conclusions can be drawn from the examination of the histogram in Fig. 3.

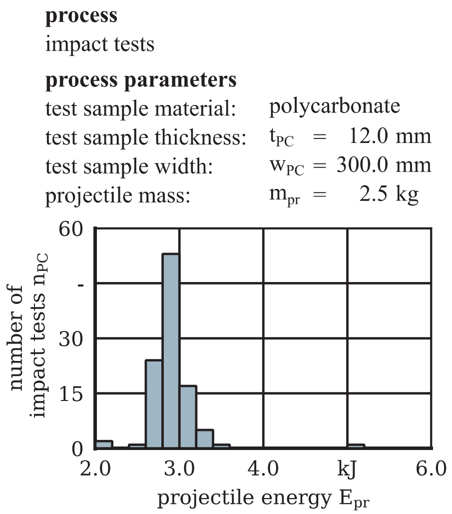

Fig. 3. Number of impact tests n_{PC} classified into 16 ranges

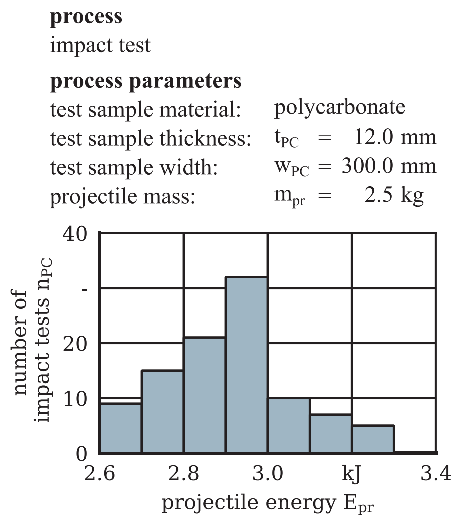

Fig. 4. Number of impact tests n_{PC} of prepared data classified into seven ranges

Conclusion 1: The aforementioned requirements for an adequate distribution fit are poorly met by the division into 16 ranges. Only for impact energies between $2.6\ kJ < E_{pr} < 3.4\ kJ$ a sufficient number of impact tests of $n_{PC} > 5$ is available. For eight ranges there is no impact test available at all. The pronounced concentration of data on impact energies on the interval of $2.6\ kJ < E_{pr} < 3.4\ kJ$ is a consequence of the bisection method used to obtain the data. It should be noted that the shape of the histogram is strongly influenced by the applied experimental method and cannot be considered randomly distributed as typically required by fitting methods.

Conclusion 2: Owing to the insufficient preconditions for an optimal distribution adjustment, it is imperative to prepare the data.

For the data preparation, all empty ranges are removed from the dataset for they do not contribute information for the distribution fitting. In addition, all ranges with an insufficient number of impact tests $n_{PC} < 5$ are also omitted. The remaining data is divided into seven new ranges, which is shown in Fig. 4. Based on this prepared dataset the impact test results can be converted from binary to qualitative data. For this, the percentage probability P_p of a failed impact test is calculated for each range. It is assumed that the probability P_p is obtained for the average projectile energy E_{pr} of the corresponding range. The results of this calculation can be found in Table 1.

Table 1. Probability P_p of a failed impact test per range

Average projectile energy E_{pr} in kJ	Number of impact tests n_{PC} per range	Probability P_p of failed impact test in %
2.65	9	11.1
2.75	15	33.3
2.85	21	23.8
2.95	32	40.6
3.05	10	80.0
3.15	7	71.4
3.25	5	80.0

4. Distribution Fitting

4.1. *Fit to normal distribution*

The probability P_p of a failed impact test can thus be employed as supporting points to fit a normal distribution. The distribution fit is performed using the software MATLAB R2018a, THE MATHWORKS INC, Natick, USA, employing a least square fit on basis of the Levenberg-Marquardt algorithm (NOCEDAL AND WRIGHT 2019). A normal distribution is characterized by its mean $\bar{x}$ and its standard deviation (STD) s. Both parameters are obtained by fitting the cumulative distribution

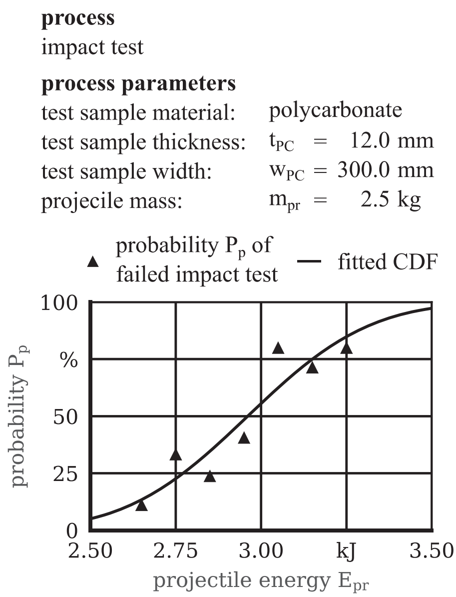

Fig. 5. Normal distribution fit

function (CDF) of a normal distribution against the probability P_p in Table 1. The result of the fitting can be found in Fig. 5 and Table 2, respectively.

Table 2. Results of the normal distribution fit

Fitting parameter	Value
Mean $\bar{x}$ in kJ	2.961
STD s in kJ	0.280
Coefficient of determination R^2	0.866

4.2. Fit to logistic distribution

The logistic distribution offers an alternative representation of the probability P for observing the failed of an impact test. A logistic distribution fitting directly employs the binary data, thus eliminating the necessity of prior data preparation and classification (Hosmer and Lemeshow 1989). A logistic function is a monotonically increasing S-shaped function, which is given by Eq. (1) (Hosmer and Lemeshow 1989).

$$p(t) = \frac{1}{1 + e^{-t}} \tag{1}$$

The variable t is a linear function of the continuous variable x, which in the present study is the projectile energy E_{pr}, such that Eq. (1), can be rewritten as shown in Eq.(2) (Hosmer and Lemeshow 1989).

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}} \tag{2}$$

The variables β_0 and β_1 are called intercept and rate parameter, respectively. Both variables are associated to the mean $\bar{x}$ and the STD s of the logistic distribution, see Eq. (3) (Hosmer and Lemeshow 1989).

$$\begin{aligned} \beta_0 &= -\bar{x}/s \\ \beta_1 &= 1/s \end{aligned} \tag{3}$$

Unlike the normal distribution fit the parameters for the logistic regression model are usually estimated by the method of maximum likelihood. Since the present study investigates the probability P of a failed impact test, a negative log-likelihood (LL) estimator is used according to Eq. (4) (Hosmer and Lemeshow 1989).

$$LL = \sum_{k=1}^{K} [y_k \ln(p_k) + (1 - y_k)\ln(1 - p_k)] \tag{4}$$

Eq. (4) represents the loss function for binary data. It measures the difference between the experimental impact test results y_k and the predicted probability p_k for the respective observation.

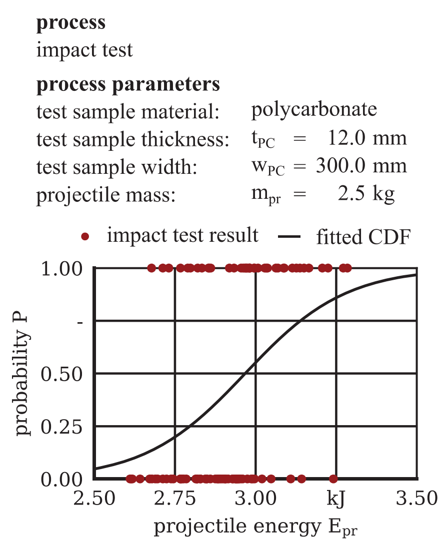

Fig. 6. Logistic distribution fit

The LL estimator maximizes the likelihood of the statistical model matching the experimental results (HOSMER AND LEMESHOW 1989). For this purpose, the optimal parameters β_0 and β_1 are determined numerically using MATLAB R2018a, THE MATHWORKS INC, Natick, USA. The fitted logistic distribution is depicted in Fig. 6, using the parameters shown in Table 3. Note, the y-axis in Fig. 6 is given in decimals rather than percentages, as this is consistent with the binary data being either "zero" or "one".

Table 3. Results of the logistic distribution fit

Fitting parameter	Value
Mean $\bar{x}$ in kJ	2.975
STD s in kJ	0.156
Coefficient of determination R^2	0.863

5. Comparison of both distributions

Fig. 7 shows a comparative plot of the normal and the logistic distribution along with the corresponding data. For the sake of convenience, the probability P for both distributions is expressed in decimal numbers. Both distributions exhibit a similar shape, with the logistic distribution showing slightly wider tails. From an inspection of the coefficient of determination R^2 it is evident that both distributions are appropriate for modelling the probability P of failed impact tests. Whereas the normal distribution has a marginally higher coefficient of determination of $R^2_{norm} = 0.866$ in comparison the logistic distribution with a coefficient of determination of $R^2_{log} = 0.863$. Based on the information provided by the two distributions, it is possible to predict the projectile energy E_{pr} associated with a specific probability P of observing a failed impact test. Table 4 shows the projectile energies E_{pr} for three selected probabilities P.

Table 4. Predicted projectile energies E_{pr} a failed impact test

Probability P for a failed impact test in %	Projectile energy E_{pr} according to: Normal distribution in kJ	Logistic distribution in kJ
1	2.31	2.26
3	2.50	2.52
10	2.60	2.63

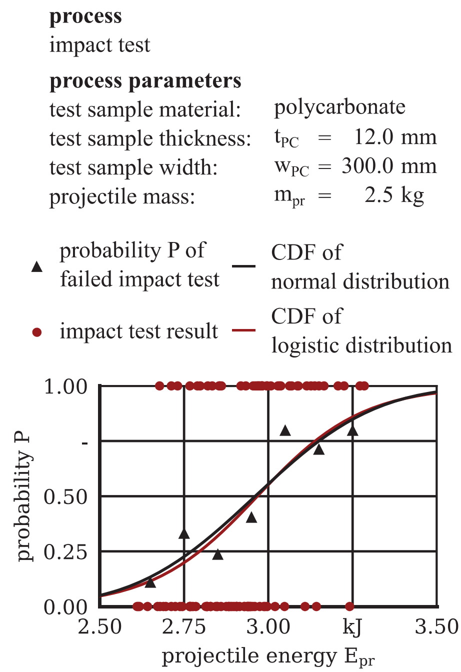

Fig. 7. Comparison of normal and logistic distribution fit

Furthermore, this probabilistic analysis allows to redefine the IR Y. Rather than characterizing it as maximum projectile energy E_{pr} a safeguard is able to withstand, it can be redefined as projectile energy E_{pr} associated with a specific probability P of failure. While the choice of such a probability P is to some extent arbitrary, safeguards for machine tools are designed to minimize the risk for operators. Therefore, in this study, the probabilistic definition of IR Y refers to a probability P = 0.01 of failure, see Eq. (5).

$$\text{IR Y} = E_{pr}\,(P = 0.01) \tag{5}$$

The shown probabilities P of failure in Table 4 indicate the logistic distribution provides a more conservative estimate of the IR Y than the normal distribution. It is important to note, however, that the dataset used for distribution fitting in this study is not randomly distributed as it was obtained through the bisection method. Therefore, the non-random distribution of the data may have an impact on the accuracy of the results obtained from the distribution fitting.

6. Conclusion and outlook

The objective of the present study was to analyze the modeling of the IR Y using a normal and a logistic distribution. For this purpose, a large dataset of impact tests on PC-sheets was evaluated. A subsequent distribution fit showed that both – the normal as well as the logistic distribution – are capable of modeling the IR Y with high accuracy. The coefficient of determination R^2 was used as a measure of the accuracy of the fit, whereas both distribution fits reach values of $R^2 > 0.86$. While both distributions are suitable for modeling the IR Y, the normal distribution requires significant data preparation. Furthermore, the normal distribution fitting is highly sensitive to the classification during the data preparation. Conversely, the logistic distribution fitting does not require any data preparation or classification of the impact test results, making it a more attractive option for modeling purposes. The dataset was obtained from UHLMANN ET AL. (2019) who investigated the effects of aging on the IR Y of PC-sheets under CL-exposure using the bisection method. Consequently, the impact test results are influenced by the method used in the original study. Thus, a thorough analysis with an appropriate experimental design is required to ensure the reliability of the results

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