Proceedings of the 33rd European Safety and Reliability Conference (ESREL 2023)
Edited by Mário P. Brito, Terje Aven, Piero Baraldi, Marko Čepin and Enrico Zio

doi: 10.3850/978-981-18-8071-1_P198-cd

Consideration of polymorphic uncertainty in model-free data-driven identification of stress-strain relations

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The method of data-driven identification, introduced by Leygue et al. (2018), enables the determination of large stress-strain data sets based on displacement fields and applied boundary conditions without postulating a specific constitutive model. The algorithm has shown to be applicable to synthetic and real data taking linear as well as non-linear material behavior into account (Dalémat et al. (2019)). The consideration of uncertain material properties by data-driven approaches, e.g. shown in Zschocke et al. (2022), leads to the requirement of data sets representing uncertain material behavior. In this contribution, different sources of uncertainty occurring within the identification of stress-strain relations are addressed and an efficient method for the identification of data sets representing uncertain material behavior based on the concept of data-driven identification is proposed. In order to demonstrate the introduced methods, numerical examples are carried out.

Keywords: Data-Driven Identification, Polymorphic Uncertainty, Data-Driven Computational Mechanics, Data Science.

1. Introduction

Data-driven methods are of increasing importance in computational mechanics. Commonly distinguished are model-based methods, aiming to approximate the constitutive material description e.g. by neural networks, and different model-free methods. The approach of material model-free data-driven computational mechanics, introduced by Kirchdoerfer and Ortiz (2016), enables to bypass any material modeling step by directly incorporating material data into the analysis. A basic prerequisite for both types of data-driven methods is a large amount of data representing the material behavior, in solid mechanics consisting of stresses and strains. Obtaining these databases numerically by multiscale approaches is computationally expensive and requires the characterization of lower scale models. In case of an experimental characterization, constitutive descriptions are generally required to compute the stress states corresponding to displacement fields.

In order to obtain realistic simulation results, uncertainty needs to be considered. Generalized polymorphic uncertainty models are utilized in order to take variability, imprecision, inaccuracy and incompleteness of material data into account by combining aleatoric and epistemic uncertainty models (Graf et al. (2014)). The objective of this contribution is the observation of possible uncertainty sources as well as the introduction of an efficient framework for uncertainty consideration in data-driven identification of stress-strain relations representing elastic material behavior.

2. Methodology

The method of data-driven identification (DDI), introduced by Leygue et al. (2018), enables the identification of stress-strain relations based on given boundary conditions and a displacement field of a test specimen, which can be obtained by digital image correlation. Based on the theoretical concept of data-driven computational mechanics, the objective of DDI is to identify material states

$(\hat{\varepsilon}_i, \hat{\boldsymbol{\sigma}}_i)$ with $i = 1, ..., n$, which are representative samples of the associated mechanical states $(\boldsymbol{\varepsilon}_e^X, \boldsymbol{\sigma}_e^X)$ of the integration points $e = 1, ..., m$. Thereby, the stress field of the considered specimen is obtained as a by-product. Required input are snapshots indicated by $X = 0, ..., N_X$ with applied forces $\boldsymbol{f}^X$, boundary conditions, a displacement field $\boldsymbol{u}^X$ as well as geometry and connectivity of the structure obtained by a finite element discretization. Associating suitable material and mechanical stets is expressed as

$$\min \sum_X \sum_e w_e^X ||(\boldsymbol{\varepsilon}_e^X - \hat{\varepsilon}_i, \boldsymbol{\sigma}_e^X - \hat{\boldsymbol{\sigma}}_i)||_{\mathbb{C}}^2 \quad (1)$$

according to a given energetic norm with the artificial stiffness $\mathbb{C}$ and integration point weights w_e as well as constrained by equilibrium and compatibility. Based on this, an equation system is derived by applying LAGRANGE multipliers η. The algorithm consists of the following steps:

(a) compute mechanical strains $\boldsymbol{\varepsilon}_e^X$,
(b) calculate mechanical stresses $\boldsymbol{\sigma}_e^X$ and η^X,
(c) set material stresses $\hat{\boldsymbol{\sigma}}_i$ as cluster centers,
(d) update mappings $\mathrm{ie}^X : (e, X) \mapsto i$,

executed iteratively until the mappings remain equal. If a random initialization of material states in the first iteration step is used, variation in the results needs to be considered. A detailed description as well as remarks concerning the individual steps are given in Leygue et al. (2018).

In this contribution, uncertain forces $\boldsymbol{f}^{X\mathrm{u}}$, resulting e.g. from measurement tolerances as well as errors arising within the conversion to applied nodal forces due to load differences, corresponding to deterministic displacement fields are considered as input. Accordingly, the desired resulting states are $(\hat{\varepsilon}_i, \hat{\boldsymbol{\sigma}}_i^{\mathrm{u}})$ and $(\boldsymbol{\varepsilon}_e^X, \boldsymbol{\sigma}_e^{X\mathrm{u}})$. Following a conventional Monte Carlo approach by repeating the analysis with samples of the uncertain input vector is not possible in this case, because different applied forces lead to different mappings and, therefore, uncertain mechanical and material strains as well. Instead, the mappings are computed based on a representative measurements of the uncertain forces analogously to the deterministic case. Based on these mappings, steps (b) and (c) are executed, whereby an uncertainty analysis is applied to compute $\hat{\boldsymbol{\sigma}}_i^{\mathrm{u}}$ and $\boldsymbol{\sigma}_e^{X\mathrm{u}}$.

3. Results and Conclusion

The introduced method is demonstrated by means of a truss example with synthetically generated input data for $N_X = 250$ snapshots and $n = 75$ desired stress-strain states. The resulting material and mechanical states with interval valued stress states are displayed in Figure 1. The obtained results are reasonable and no over- or underestimation of the desired uncertainty is visible.

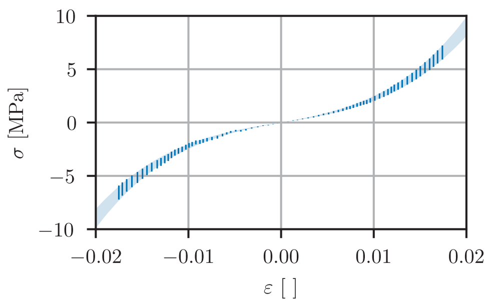

Fig. 1. Material states obtained by the introduced approach and uncertain reference constitutive description.

Acknowledgement

The research activities result from the project "Structural Design of Reinforced Concrete based on Multiscale Modelling and Polymorphic Uncertainty" within the DFG Priority Program SPP 1886.

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