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Quantum Optimization for Redundancy Allocation Problem Considering Various Subsystems

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The Redundancy Allocation Problem (RAP) is a widely studied NP-hard Combinatorial Optimization (CO) issue in reliability engineering. It involves assigning components to parallel or serial subsystems to maximize system reliability within a given budget. RAP has been extensively researched in various fields, such as electrical power systems and computer networks, with different configurations, including multi-objective, bi-objective, and mono-objective setups, as well as series, parallel, and parallel-series equipment arrangements. In recent years, quantum computing has emerged as a promising approach for solving CO problems, including RAP. Quantum processors, such as those developed by D-Wave Systems, have undergone significant research and testing in academic and commercial environments for solving combinatorial problems. This study aims to solve two multi-subsystems RAP instances using a D-Wave quantum annealing computer. The results provide a concept proof of the usability of quantum hardwares and hybrid-quantum algorithms for RAP.

Keywords: Redundancy Allocation Problems. Reliability Optimization. Quantum Optimization. Quantum Computing.

1. Introduction

The Reliability Allocation Problem (RAP) involves assigning redundant components to subsystems to maximize system reliability. This problem has multiple applications from both a multi-objective and mono-objective perspective and can be solved using various exact and metaheuristic methods. To effectively handle combinatorial optimization problems such as RAP, several Ising model-based computers, also known as annealing machines, have been developed (Ajagekar et al., 2019). In this study, we aim to solve the RAP problem using a Hybrid Quantum Annealing quantum method, via D-Wave computer. In a previous study (Araújo et al., 2022), we solved the RAP problem with only one system and multiple components. In this study, we consider multiple subsystems and propose an approximative formulation that fits the Quadratic Unconstrained Binary Optimization (QUBO) problem.

2. Problem Formulation

The mono-objective problem aims to maximize system reliability, subject to a cost constraint (C) for purchasing components. The system consists of different component types (ct) organized into subsystems (j), and each subsystem must have components within a minimum ($n_{j,min}$) and a maximum ($n_{j,max}$) limit.

A single subsystem failure in a series-parallel configuration can cause the entire system to fail. The reliability of the entire system is limited by the least reliable subsystem. While minimizing the probability of failure of the least reliable subsystem can increase system reliability. However, it can also compromise the reliability of other subsystems if done excessively. Therefore, it is important to balance the failure probabilities of all subsystems while minimizing the probability of failure of the least reliable subsystem.

The objective function in this approach, Eq. (1), involves the natural logarithm of the product of subsystems' failure probability ($\prod_{j=1}^{s} F_j$). and the natural logarithm of the maximum subsystem failure probability ($\mathrm{lF_{max}} = \ln(\mathrm{F_{max}})$). To prioritize the search for systems with a low probability of failure in critical subsystems, the factor s^2 (i.e., the squared number of subsystems) has been adopted for $\mathrm{lF_{max}}$. In Eq. (2), the right-hand side is equal to $\ln(\mathrm{F_j})$.

$$\min\ s^2 \cdot \mathrm{lF_{max}} + \sum_{j=1}^{s} \sum_{k=1}^{ct_j} x_{jk} \ln(1 - R_{jk}) \quad (1)$$

$$\text{s.a., } \mathrm{lF_{max}} \geq \sum_{k=1}^{ct_j} x_{jk} \ln(1 - R_{jk}),\ \forall j \quad (2)$$

$$\sum_{j=1}^{s} \sum_{k=1}^{ct_j} c_{jk} x_{jk} \leq C \quad (3)$$

$$n_{j,min} \leq \sum_{k=1}^{ct_j} x_{jk} \leq n_{j,max},\ \forall j \quad (4)$$

$$x_{jk} \in \{0, 1, \ldots, n_{j,max}\}, \forall j;\ \forall k \quad (5)$$

We utilized the D-Wave library to convert the problem, modeled as a QUBO, into an Ising Hamiltonian. The decision variables were converted into binary; inequality constraints were transformed into equality constraints by adding slack and excess variables; and these constraints were then incorporated into the objective function as a penalty, resulting in the QUBO formulation. To solve the problem, we employed the Leap Hybrid CQM Sampler from D-Wave's Ocean SDK, which is a hybrid classical-quantum solver that combines a classical optimizer with a quantum sampler. The classical optimizer used by default is the L-BFGS-B algorithm.

4. Results

We tested small instances for analysis and to demonstrate the effectiveness of the hybrid-quantum algorithm. It should be noted that due to qubit limitations, we were unable to test quantum simulators provided by IBM, such as the Statevector and QASM simulator, as we did in the previous study with only one subsystem (Araújo et al., 2022).

The instances used for testing are in Table 1. In instance #1, the hybrid-quantum approach found the optimal solution with an energy value of -8.11. The results showed that subsystem 1 had one unit of component 1, while subsystem 2 had two components (one of type 1 and the other of type 2). The overall system reliability resulted in 0.873. For instance #2, the best solution was at an energy level of -13.86. The reliability analysis showed that at subsystem 1, there was one unit of component 1 and one unit of component 3. At subsystem 2, there was one unit of component 1 and one unit of component 2. At subsystem 3, there were two units of component 1. Overall, the system reliability reached 0.9126.

Table 1. Instances of the multi-subsystem active parallel RAP ($n_{j,min}$ = 1, for s = 1,..., s).

#	s	$n_{j,max}$	C	ct_j	R_{1k}	R_{2k}	R_{3k}	c_{1k}	c_{2k}	c_{3k}
1	2	2 2	8	2 2	0.9 0.7	0.9 0.7	-	3 2	3 2	-
2	3	3 2 2	13	3 2 2	0.9 0.8 0.7	0.9 0.8	0.8 0.7	3 2 1	3 2	2 1

It is important to emphasize that all these outcomes were equal to those acquired by the exhaustive method, and they were obtained in less than 20 seconds. Note that, as stated earlier, we tested small instances (detailed in Table 1) that, even though NP-hard in nature, were able to map in reasonable time to feasible solutions through the exhaustive method. In future research, more instances will be tested, and we intend to perform experiments with algorithms that utilize quantum circuits, such as the QAOA and the VQE.

5. Conclusion

The field of combinatorial optimization has been greatly influenced by quantum computing in recent years. This study adds to this trend by presenting a linear problem approximation of a multi-subsystem series-parallel RAP, which is solved using a hybrid-quantum approach. It should be noted that while there are limitations in terms of available qubits, this technique shows promise for future reliability optimization research.

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