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Flame Front Spread Model of Forest Fire in Steep Canyon Terrain

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Against the backdrop of global climate change, unusually intense and widespread forest fires are becoming more common, leading to significant human fatalities, as well as socioeconomic and ecological losses. In Europe alone, the damage inflicted by wildfires in 2022 is estimated to be at least €2 billion. To prevent and suppress forest fires effectively, it is crucial to understand their propagation behavior, especially in steep-sloped canyons where the risk of rapid spread is high. However, there has been limited research on this topic. In this study, we conduct a mathematical analysis of the existing gentle slope canyon model to improve our understanding of steep-slope canyon fires' behavior and enhance the accuracy of fire spread models. Our model is validated through publicly available experimental data, which demonstrates its accuracy. The derived fire spread model has practical implications for forest fire protection measures and fire suppression strategies in canyon terrain, helping to reduce the risk and impact of forest fires in the future.

Keywords: Forest fire, Steep canyon, Fire protection, Fire suppression.

1. Introduction

Forest fires, also known as wildfires, have been a natural phenomenon throughout history. While these fires play an important role in shaping the ecology of forests, they can also have devastating effects on human communities. With the impact of climate change, the frequency and severity of forest fires are increasing (Burrell et al. 2022).

In 2012, a study found that most EU countries experienced a considerable excess of forest area affected by fire (Gutiérrez and Lozano 2013). In 2022 alone, the economic damage inflicted by wildfires in Europe is estimated to be at least €2 billion (Meier, Elliott, and Strobl 2023). The Australian wildfire season of 2019/2020 resulted in the largest recorded loss of houses and land. It is estimated that over 1 billion animals were killed, with some species facing the threat of extinction (Filkov et al. 2020). The economic damage from the fires is estimated to be around USD 1.5-2 billion (Guha-Sapir, 2021; Bevere, 2021). In 2019, a total of 50,477 wildfires occurred in the United States, affecting 1.9 million hectares of land, an area approximately 7 times the size of New York City (Frank, 2020). As the threat of forest fires continues to increase, it is now more critical than ever to understand their behavior and develop effective strategies for managing and mitigating their impacts.

One key factor affecting forest fire behavior is the topographical environment (Viegas and Pita 2004). In 2019, a forest fire broke out in the Liangshan valley region of Sichuan, China. Due to an abrupt change in wind direction, the topographical environment contributed to the fire's deflagration, which tragically led to the deaths of 27 forest firefighters and 4 residents (Zhang et al. 2021). Mountain fire deflagrations, which are characterized by rapid fire propagation and a high energy release rate, are often cited as a significant cause of fatalities during forest fires (Viegas and Simeoni 2011). Research has shown that it is difficult for a fire to maintain a steady burn rate in a canyon and fire deflagrations are more likely to occur in narrow canyons due to the topography of mountainous regions (Viegas 2006). Additionally, when the canyon is steep, it has a higher potential to burst into deflagration within a short period of time (Viegas and Pita 2004).

Limited research has explored the behaviour of forest fires in canyon topography. In 2004, Viegas and Pita conducted an experiment using sloping fuel beds to simulate and better understand the spread of canyon fires (Viegas and Pita 2004). Their study combined experimental and analytical methods to propose a double elliptical model for the spread of canyon fires. The model considers a symmetrical canyon with uniform vegetation, a flat surface, and a single symmetrical ignition without the effects of wind. The model accurately predicts the flame spread rate on gentle slopes. However, when the simulated canyon slope angle increases, it was observed that the head fire deviates from the maximum slope direction and tends to approach the canyon centerline axis, resulting in the flame evolving into two triangles instead of a double ellipse. The fire spread along the canyon centerline axis tends to equal or even surpass the progress along the maximum slope direction which cannot be accounted for by the proposed model. Viegas and Pita attributed this behavior to the feedback effects of fire convection on the response zone.

This paper aims to improve our understanding of canyon fires' behavior and enhance the accuracy of fire spread models in steep-sloped canyons. By conducting a mathematical analysis of flame propagation across different slope conditions, we aim to address the limitations of the existing double elliptical forest fire propagation model in accurately predicting outcomes on steep slopes. The proposed enhanced model is then validated through publicly available experimental data in the literature.

The findings of this study will contribute to the development of more accurate and effective fire spread models for steep-sloped canyons, which will be useful for forest managers and firefighters in developing effective fire suppression and prevention strategies.

2. Methodology

2.1. *Rate of spread (ROS)*

The rate of spread (ROS) of a forest fire refers to the speed at which the fire is spreading across the landscape, measured in feet or meters per minute, hour, or day. It is an important metric used by fire managers and firefighters to assess the behavior and potential impact of a wildfire. To determine the rate of spread (ROS) of a forest fire, it is necessary to consider the energy emitted at the fire front with the energy required to ignite the fuel in the fire area. The ROS value is predominantly influenced by the type of fuel bed, the slope of the terrain, and the nearby wind flow, in addition to the convective flow generated by the fire itself. When a fire spreads on a slope or into a favourable wind direction, it creates a head with a higher ROS in comparison to the rest of the fire parameters. To assess the effect of topography on the ROS of a forest fire, it is assumed that other factors, such as fuel bed properties and variables that change over time (such as ambient wind speed and direction), remain constant. Additionally, to simplify calculations, vectors such as flow velocity and ROS are assumed to be parallel to a common direction, making them independent components that can be used as scalars in equations.

2.2. *Geometry of a canyon topography*

Canyons are a commonly occurring topographic feature in complex terrains. They are deep, narrow valleys or gorges that are typically formed by the erosion of rock by water or wind over millions of years. In this study, we assume that the canyon's surface is flat and has no curvature.

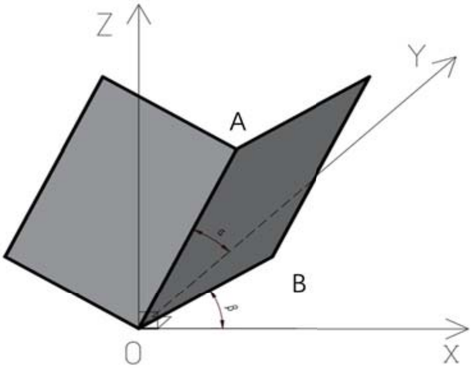

Fig 1. Definition of the geometry of the canyon.

Thus, we define a canyon as an area with two sloping surfaces and a horizontal plane. As depicted in Figure 1, the shaded regions represent the slopes on both sides of the canyon, while the plane XOY represents the horizontal surface. The angle formed between the line segment OA and the horizontal plane, where the slopes intersect, is denoted by α. Similarly, the angle formed between the line segment OB and the horizontal plane is denoted by β.

2.3. Fire spread model for gentle slope (α<30°)

To derive the steep slope fire propagation model, we need to revisit the original model developed by Viegas and Pita (Viegas and Pita 2004). Their experiments showed that when the slope is less than 30 degrees, the flame eventually evolves into a double ellipse, as shown in Figure 2. In this model, we use four characteristic values to describe the flame propagation: S_1, S_2, S_3, and S_4, which are defined in Table 1.

Table 1. Nomenclature of characteristic values

Designation	Definition
O_{ig}	Ignition point
S_1	Fire spread distance along y axis
S_2	Fire spread distance along y_s axis
S_3	Fire spread distance along x_s axis
S_4	Horizontal fire spread distance
R	Rate of spread (ROS) along the canyon centerline (m/s)
R_0	Rate of spread of a linear flame front in the absence of slope (m/s)
R_h	Rate of spread of fire head (m/s)
x_s	Major axis of the ellipse
y_s	Minor axis of the ellipse
θ	The Angle between x_s and the horizontal direction

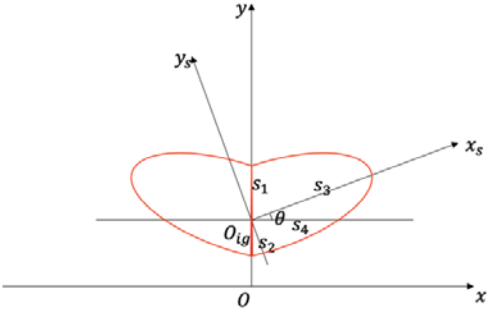

Fig 2. Schematic of an analytical model of a gentle slope canyon fire (α < 30°)

For a symmetrical canyon, we have

$$S_2 = \int_0^t R_0 dt \tag{1}$$

And

$$S_3 = \int_0^t R_h \, dt \tag{2}$$

R_0 is the ROS of the linear flame front under zero-slope conditions, and R_h is the ROS of the fire head along the elliptical principal axis.

Rotate the coordinate axis xOy until the x-axis coincides with the main axis of the ellipse and the origin O coincides with the ignition point O_{ig} to obtain the new coordinate system x_sOy_s. For the rotated coordinate system x_sOy_s, we have

$$\begin{cases} x_s = xcos\theta + ysin\theta \\ y_s = -xsin\theta + ycos\theta \end{cases} \tag{3}$$

The analytic equation of the ellipse is

$$\frac{x_s^2}{s_3^2} + \frac{y_s^2}{s_2^2} = 1 \tag{4}$$

By combined use of (Eq.3) and (Eq.4), we have

$$s_2^2 x_s^2 + s_3^2 y_s^2 = s_2^2 s_3^2$$

$$s_2^2(xcos\theta + ysin\theta)^2 + s_3^2(xsin\theta - ycos\theta)^2 = s_2^2 s_3^2$$

To get S_1, assume $x = 0$

$$S_1 = y = \frac{S_2 S_3}{\sqrt{s_2^2 sin^2\theta + s_3^2 cos^2\theta}}$$

To get S_4, assume $y = 0$

$$S_4 = x = \frac{S_2S_3}{\sqrt{s_2^2cos^2\theta + s_3^2sin^2\theta}}$$

Now we have four characteristic parameters

$$\begin{cases} S_1 = \dfrac{S_2S_3}{\sqrt{s_2^2sin^2\theta + s_3^2cos^2\theta}} \\ S_2 = \displaystyle\int_0^t R_0dt \\ S_3 = \displaystyle\int_0^t R_h\,dt \\ S_4 = \dfrac{S_2S_3}{\sqrt{s_2^2cos^2\theta + s_3^2sin^2\theta}} \end{cases}$$

By combined use of S_1, S_2 and S_3, we can get

$$R = \frac{dS_1}{dt} = \left(\frac{S_2S_3}{\sqrt{s_2^2sin^2\theta + s_3^2cos^2\theta}}\right)'$$

$$= \frac{(R_0S_3 + R_hS_2)\sqrt{s_2^2sin^2\theta + s_3^2cos^2\theta} - \frac{1}{2}S_2S_3\frac{2sin^2\theta R_0S_2+2cos^2\theta R_hS_3}{\sqrt{s_2^2sin^2\theta+s_3^2cos^2\theta}}}{(s_2^2sin^2\theta + s_3^2cos^2\theta)^{3/2}}$$

$$= \frac{sin^2\theta R_hS_2^3 + cos^2\theta R_0S_3^3}{(s_2^2sin^2\theta + s_3^2cos^2\theta)^{3/2}}$$

The equation demonstrates that the ROS is influenced by the parameters Angle θ, R_h, R_0, S_2 and S_3. In the real-life scenarios, these parameters can be directly measured. When the slope Angle α is equal to 0, the whole fuel bed can be regarded as lying flat on the ground, where $\theta = 90°$, $R = R_h$ is obtained. As per the model, when the flame propagates on a flat surface, the spread rate is equivalent to the rate of the head fire. This agrees with the actual spread behavior of flames.

The original model developed by Viegas and Pita was presented in the form of Eq.5, containing the tangent function (Viegas and Pita 2004). In this study, the model was modified to represent the final results in terms of sine and cosine functions. This modification allows us to use the same variables to model canyon slope angles greater than 30 degrees, improving the consistency of the results.

$$R = \frac{S_2^3R_h+S_3^3tan^2\theta R_0}{cos\theta\cdot(S_2^2+S_3^2tan^2\theta)^{3/2}} \tag{5}$$

3. Results & Validation

3.1. *Fire spread model for steep slope* $(30° < \alpha < 90°)$

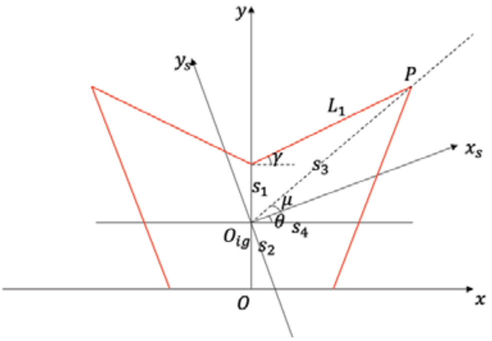

Fig 3. Schematic of an analytical model of a steep slope canyon fire (30° < α < 90°)

The experiments conducted by Viegas and Pita in 2004 (Viegas and Pita 2004) demonstrated that the behavior of flame propagation changes when the slope angle exceeds 30 degrees. Instead of a double-ellipse, the flame takes the form of two triangles, as depicted in Figure 3. To derive the steep slope model, it is necessary to introduce two additional angles, μ and γ, which are illustrated in Figure 3 for plane P and defined in Table 2.

Table 2. Nomenclature of characteristic values

Designation	Definition
θ	The Angle between x_s and the horizontal direction
μ	The Angle between line $O_{ig}P$ and x_s
γ	The Angle between L_1 and the horizontal direction

To describe the evolution of the flame front, we defined four distances S_1, S_2, S_3 and S_4 with reference to Figure 3. Unlike the previous elliptic model, which assumes an elliptic flame shape, it is challenging to use a geometric function to describe the flame propagation when it spreads as a triangle. Therefore, we focus the model on the flame front by first determining the flame contour range, as shown in Figure 2. The flame front is assumed to be straight line L_1 . Assuming that L_1 goes through point P and is represented by the equation $L_1\colon y = tan\gamma x + b$, we can derive an equation for L_1 as follows:

$$tan\gamma \cdot S_3 \cdot \cos(\mu+\theta) + b = S_3 \sin(\mu+\theta)$$

$$b = S_3(sin(\mu+\theta) - tan\gamma cos(\mu+\theta))$$

$$b = S_3\left(\frac{sin(\mu+\theta)cos\gamma - sin\gamma cos(\mu+\theta)}{cos\gamma}\right)$$

$$b = \frac{\sin(\mu+\theta-\gamma)}{cos\gamma} S_3$$

Therefore, the equation for L_1 becomes:

$$L_1\colon y = tan\gamma x + \frac{\sin(\mu+\theta-\gamma)}{cos\gamma} S_3$$

To get S_1, we can assume $x = 0$, which gives:

$$S_1 = y = \frac{\sin(\mu+\theta-\gamma)}{\cos\gamma} S_3$$

Combining the equations for S_1 and S_3, we get the equation for the rate of spread, R, as:

$$R = \frac{dS_1}{dt} = \frac{\sin(\mu+\theta-\gamma)}{\cos\gamma} R_h \qquad (6)$$

The equation shows that ROS is dependent only on the three characteristic angles (μ, θ, γ) and R_h when the slope is greater than 30 degrees. Thus, under steep slope conditions, the spread speed and direction of the flame can be determined by measuring the three characteristic angles. The R_h value can be predetermined and measured in the laboratory based on local vegetation, temperature, humidity, and other relevant variables.

3.2. *Model validation*

To verify the accuracy of our model, we compared our calculated results with the experimental data obtained from previous studies conducted by Viegas and Pita (Viegas and Pita 2004) and Xie et al. (Xie et al. 2020). Specifically, we obtained experimental data for a slope angle of 30° from these studies, which we used to compare with our model's predicted results.

In both experiments, the researchers used sloping fuel beds to simulate forest fires in canyon terrain under indoor windless conditions. Both experiments assume that the terrain surface is a flat surface, i.e. without any curvature and used dead pine needles as fuel. Using this experimental setup, forest fire scenarios can be scaled down effectively. Digital cameras were used to capture test images and infrared cameras were used to extract fire lines.

The obtained experimental data is shown in Table 2, which includes variables $\mu+\theta$, γ, R_h, and R. Both sets of data were measured under the condition α= 40°, β= 30°. We substituted the values into Eq. (6) to calculate R for both data sets.

Table 2. Data sets (α=40°, β=30°)

Variables	Data set 1 (Viegas and Pita 2004)	Data set 2 (Xie et al. 2020)
$\mu+\theta$	53.9°	64.2°
γ	18°	23°
R_h	$1.67m/s$	$1.82m/s$
R	$0.97m/s$	$1.26m/s$

When we substitute the experimental values into (Eq.6), we get

$$Data\ set\ 1\colon R = \frac{\sin(\mu+\theta-\gamma)}{\cos\gamma} R_h = \frac{\sin(53.9° - 18°)}{\cos 18°} * 1.67 = 1.03m/s$$

$$Data\ set\ 2\colon R = \frac{\sin(\mu+\theta-\gamma)}{\cos\gamma} R_h = \frac{\sin(64.2° - 23°)}{\cos 23°} * 1.82 = 1.30m/s$$

The ROS values calculated from the model were 1.03 m/s for data set 1 and 1.30 m/s for data set 2. The actual ROS values measured by the researchers were 0.97 m/s (Viegas and Pita, 2004) and 1.26 m/s (Xie et al., 2020), respectively. By comparing our model results with the experimental data, we found that the errors were 6% and 3%, respectively, indicating a good agreement between the model and the real-world conditions. These findings demonstrate the usefulness of our model in predicting ROS values for steep-slope fire under real-world conditions.

5. Discussion

This article proposes a flame propagation model for steep slopes that reflects real-world canyon topography conditions. The existing double-ellipse model is inadequate in describing flame spread on steeper slopes. Our proposed model takes the form of two triangles, which considers the effect of the uphill wind generated by the flames tilting the front edge of the flame towards the fuel bed. As the slope angle increases, the interaction between flames becomes more significant, causing the flame to stretch and increase in length, and generating a stronger radiation heat source. This deviation from the maximum slope line of the canyon is a critical factor that our model addresses.

The proposed model reveals that reducing the rate of spread of the flame front, i.e., R_h, is key to reducing ROS. This can be achieved by reducing air convection and flammable material at the front of the flame. Our model has significant practical implications for fire safety, as it allows firefighters to predict the rate and direction of flame spread, optimizing suppression strategies, and reducing the risk of fatalities and property damage. For example, firefighters can be deployed at safer locations away from potential flame fronts. At the same time, the fire spread model can be used to create fire management plans for specific forests or areas, such as specific high slope areas in canyons. Firefighters can simulate possible fires in advance in the laboratory and create specialised plans. These plans can be used to identify areas of high fire risk, prioritise resources and determine the most effective fire suppression and prevention strategies. Furthermore, fire spread models can provide estimates for how long a fire is likely to impact the forest and nearby communities, which can aid in the development of evacuation plans and allocation of resources to mitigate the effects of the fire.

However, the proposed model has limitations. It does not consider the effects of other parameters such as vegetation cover, temperature, humidity, fuel moisture content, and wind speed. Including these parameters may render the model too complex for practical use. Obtaining accurate and reliable data on these variables may also be challenging, as canyon regions are typically remote and inaccessible. Meanwhile, our study quantified the effect of the slope α on fire propagation, assuming that beta remained constant, while acknowledging that the Angle β may also impact combustion conditions. Furthermore, the model validation section could be further enhanced if additional combustion data were available for conditions where slope α exceeds 40 degrees. Future studies may refine the proposed model by including these parameters and addressing these challenges.

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