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An Adaptive Prescriptive Maintenance Policy for a Gamma Deteriorating Unit

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In this paper, we propose an adaptive prescriptive maintenance policy that generalizes one recently proposed in the literature. The policy consists in performing a single inspection aimed at measuring the degradation level of the unit at a predetermined inspection time. Based on the outcome of this inspection, it is decided whether to immediately replace the unit or to postpone its replacement to a later time. In case of postponement, the usage rate of the unit may be changed if deemed convenient. The main novelty of the proposed policy is that, in case the replacement is postponed, the value of the usage rate in the remainder of the maintenance cycle (i.e., the time elapsing between the inspection time and the replacement time) is determined based on the measured degradation level at the inspection time. The optimal maintenance policy is defined by maximizing the long-run average utility rate. After each replacement the unit is considered as good as new. The lifetime of the unit is defined by using a failure threshold model. It is assumed that failures are not self-announcing and that failed units can continue to operate, albeit with reduced performance and/or additional costs.
Maintenance costs are computed considering the cost of preventive replacements, corrective replacements, inspections, logistic costs, downtime costs (which account for time spent in a failed state), and costs that account for the change of the unit working rate. These latter costs also include the possible penalty determined by failure to comply with contract clauses.

Keywords: gamma process, prescriptive maintenance, adaptive maintenance policy, renewal reward theorem.

1. Introduction

The main role of maintenance planning in industrial applications is to guarantee reliable and safe functioning of equipment. However, given that it often entails disruption to normal operations (for example, to perform in-depth inspections) it is typically perceived as a time-intensive and costly task.
On the other hand, maintenance is also an activity that could create opportunities for improvement.
In the search for maximum operational efficiency, modern maintenance strategies such as prescriptive maintenance have been proposed as a potentially effective tool.

Although a large number of papers dealing with the concept of prescriptive maintenance have been proposed in the literature, a formal and widely accepted definition that clearly distinguishes prescriptive maintenance from other classical strategies (such as predictive maintenance and condition-based maintenance) is still missing. Nevertheless, Iung (2019) suggested that prescriptive maintenance differs from other approaches because maintenance recommendations (i.e., the prescriptions) are defined by taking into account all functionalities of a system.

Longhitano et al. (2021) further stressed that prescriptive recommendations go beyond describing what, when, and how to perform maintenance, but also provide precise operative instructions on how to adjust the system operating conditions to reach a desired outcome.

Drawing inspiration from modern prescriptive maintenance ideas, Esposito et al. (2022) proposed a maintenance policy where, based on a single inspection, performed at a predetermined inspection time, by using a condition-based rule, it is decided whether to immediately replace the unit or to adjust its usage rate to a predetermined value and postpone its replacement to a future time.

In this paper, we suggest a prescriptive policy that generalizes the one proposed by Esposito et al. (2022) by assuming that, in case the replacement is postponed, the new usage rate can be adaptively determined based on the measured degradation level at the inspection time.

As other policies where a single inspection is allowed over the lifetime of the unit (e.g., see Esposito et al. (2021), Finkelstein et al. (2020), Cha et al. (2022a), Cha et al. (2022b)), this strategy suits experimental situations where inspections are very costly (Alaswad and Xiang (2017)). Moreover, in the cases where the time intervals between successive maintenance interventions are subjected to constraints (for example, limited maintenance personnel availability, contractual clauses, etc…) then the possibility of changing the usage rate can introduce another degree of freedom and provide a better tradeoff between preventive replacement, corrective replacement, inspection, and operational costs.

The performance measure adopted to define the optimal policy is the long-run average maintenance utility rate.

The remainder of the paper is structured as follows. Section 2 illustrates the proposed maintenance policy. Section 3 is devoted to the description of the adopted degradation model. Sections 4 and 5 present the cost model and the formulation of the long-run average maintenance utility rate. Section 6 reports the results of an example of application of the policy. Section 7 provides some conclusions.

2. The maintenance policy

In this paper, we focus on a single-unit system that, during its operating life, degrades gradually over time according to a gamma degradation process.

We assume that the unit failure is determined by the first (and sole) passage time of its degradation level to a fixed threshold (say w_M), that after this "soft" failure (Meeker and Escobar (1998)) the unit can continue operating until its replacement, though with reduced performances and/or additional costs, and that failures are not self-announcing.

Accordingly, we suppose that failures can only be detected through costly ad hoc, non-destructive inspections, which allow to observe the exact degradation level of the units and that corrective and preventive replacements can be performed only at the maintenance epochs.

As in Esposito et al. (2022), the proposed policy consists in performing a single inspection at a predetermined time τ whose outcome is a measurement of the current degradation level of the unit (hereinafter denoted as w_τ). This measurement is used to decide, via a condition-based rule, whether to immediately replace the unit or to postpone its replacement to time 2τ (no additional inspection will be performed at 2τ). In case the replacement is postponed, if it is deemed economically convenient, the usage rate for the remainder of the operating life of the unit can be changed within predetermined limits, according to convenience, influencing both the future evolution of the degradation process and the operating costs.

The main novelty with respect to Esposito et al. (2022) is that, here, in case the replacement is postponed the new usage rate is determined based on the adaptive rule described in Table 1, where $L_1 < L_2 < \cdots < L_k \leq w_M$, $u_1 > u_2 > \cdots > u_k$ denote the usage rates that (based on w_τ) are used in the next time interval, and L_k is the preventive replacement threshold.

At the beginning of the maintenance cycle the usage rate is set to u_0. As in Esposito et al. (2022), we suppose that also u_0 is a decision parameter, which should be set on the basis of economic considerations.

The unit is replaced either preventively or correctively based on its state at the replacement time. Replacements are assumed to be equivalent to a perfect repair, which restores the unit to an "as good as new" state. Consequently, the time between two replacements can be thought of as a cycle of a renewal process.

The components of the vector $\boldsymbol{\xi} = \{L_1, \ldots, L_k, u_0, u_1, \ldots, u_k\}$ should be intended as design parameters. The value of $\boldsymbol{\xi}$ which maximizes the long-run average maintenance utility rate and defines the optimal policy is denoted by $\boldsymbol{\xi}^* = \{L_1^*, \ldots L_k^*, u_0^*, u_1^*, \ldots, u_k^*\}$.

The number of classes k is envisaged as a "hyperparameter" that should be set a priori with the aim of finding a satisfactory tradeoff between performance of the policy and computational burden, which both increase with k.

Table 1. Adaptive condition-based rule adopted to perform decision-making.

Degradation level measured at τ	Decision
$w_\tau > L_k$	Replace immediately
$L_{k-1} < w_\tau \le L_k$	Postpone replacement to 2τ, set usage rate to u_k
⋮	⋮
$L_1 < w_\tau \le L_2$	Postpone replacement to 2τ, set usage rate to u_2
$w_\tau \le L_1$	Postpone replacement to 2τ, set usage rate to u_1

It is worth remarking that the policy obtained by setting $k = 1$ coincides with the one proposed in Esposito et al. (2022).

Table 2, where $w_{2\tau}$ is the degradation level at time 2τ, lists all the possible scenarios along with the corresponding maintenance actions to be taken and the length of a maintenance cycle $T(w_\tau)$.

Table 2. Possible scenarios and corresponding maintenance action and cycle length.

Experimental scenario	Maintenance action	Cycle length $T(w_\tau)$
$L_k < w_\tau \le w_M$	Preventive replacement at τ	τ
$w_\tau > w_M$	Corrective replacement at τ	τ
$w_\tau \le L_k$ and $w_{2\tau} \le w_M$	Preventive replacement at 2τ	2τ
$w_\tau \le L_k$ and $w_{2\tau} > w_M$	Corrective replacement at 2τ	2τ

Note that, despite the notation not highlighting it, $T(w_\tau)$ functionally depends on $\boldsymbol{\xi}$.

3. Degradation model

The gamma process $\{Y(t), t \ge 0\}$ is a monotonic increasing stochastic process with gamma distributed independent increments. To fully define it is necessary to specify an initial condition (here $Y(0) = 0$) and the probability density function (pdf) of its generic increment $\Delta Y(t_1, t_2)$:

$$f_{\Delta Y(t_1,t_2)}(\delta) = \frac{\delta^{\Delta\eta(t_1,t_2)-1}}{\theta^{\Delta\eta(t_1,t_2)} \cdot \Gamma\big(\Delta\eta(t_1,t_2)\big)} \cdot e^{-\frac{\delta}{\theta}}, \quad \delta \ge 0,$$

where $t_2 > t_1 \ge 0$, θ $(\theta > 0)$ is the scale parameter, $\Delta\eta(t_1, t_2) = \eta(t_2) - \eta(t_1)$, $\eta(t)$ is the age function (a non-negative, monotone increasing function), and $\Gamma(\cdot)$ is the complete gamma function.

In this paper, we assume that the degradation process $\{W(t), t \ge 0; \boldsymbol{u}(t)\}$ of the considered unit, given the history of the usage rate $\boldsymbol{u}(t) = \{u(y), 0 \le y \le t\}$ up to and including the time t, can be modeled by a gamma process whose increment $\Delta W(t, t + dt)$ over the elementary time interval $(t, t + dt)$ is gamma distributed with conditional pdf:

$$f_{\Delta W(t,t+dt)}\big(\delta; \boldsymbol{u}(t)\big) = f_{\Delta W(t,t+dt)}\big(\delta; u(t)\big) = \frac{\delta^{\eta'(t;u(t))dt-1} \cdot e^{-\frac{\delta}{\theta}}}{\theta^{\eta'(t;u(t))} \cdot \Gamma\big(\eta'(t;u(t)) \cdot \mathrm{d}t\big)}, \quad \delta \ge 0. \quad (1)$$

From the pdf (1) it also follows that, given $u(t)$, the increment $\Delta W(t, t + dt)$ is independent of $W(t)$ and of the past history of the usage rate $\boldsymbol{u}(t^-) = \{u(y), 0 \le y < t\}$.

Moreover, as in Tseng et al. (2009), the distribution of the increment $\Delta W(t, t + dt)$ depends on $u(t)$ only through the value of its shape parameter $\eta'\big(t; u(t)\big) \cdot dt$, while the scale parameter θ is independent of the usage rate.

The derivative of the age function $\eta'\big(t; u(t)\big)$ must be positive and integrable with respect to t. In this paper, we adopt the commonly used power-law expression $\eta'\big(t; u(t)\big) = a\big(u(t)\big) \cdot b \cdot t^{b-1}$.

Based on this modeling solution, under the maintenance policy described in Section 2 the degradation level $W(t)$ at the time t, for any $t \le \tau$ (i.e., at any time before the first inspection), is distributed as a gamma random variable with pdf:

$$f_{W(t)}(w; u_0) = \frac{w^{\eta(t;u_0)-1}}{\theta^{\eta(t;u_0)} \cdot \Gamma\big(\eta(t;u_0)\big)} \cdot e^{-\frac{w}{\theta}}, \quad w \ge 0 \quad (2)$$

and cdf:

$$F_{W(t)}(w; u_0) = \frac{\gamma\left(\eta(t;u_0), \frac{w}{\theta}\right)}{\Gamma\big(\eta(t;u_0)\big)}, \quad w \ge 0 \quad (3)$$

where:

$$\eta(t; u_0) = \int_0^t \eta'\big(y; u(y)\big) \cdot \mathrm{d}y = \int_0^t a\big(u(y)\big) \cdot b \cdot y^{b-1} \cdot \mathrm{d}y = a(u_0) \cdot t^b.$$

and $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function.

Similarly, given $u_h\ (h = 1, \dots, k)$, the degradation increment $\Delta W(\tau, t)$ for any $t > \tau$ (i.e., at any time after the inspection time) is gamma distributed with conditional pdf:

$$f_{\Delta W(\tau,t)}(\delta; u_h) = \frac{\delta^{\Delta\eta(\tau,t;u_h)-1} \cdot e^{-\frac{\delta}{\theta}}}{\theta^{\Delta\eta(\tau,t;u_h)} \cdot \Gamma\left(\Delta\eta(\tau, t; u_h)\right)},\ \ \delta \geq 0, \quad (4)$$

and conditional cdf:

$$F_{\Delta W(\tau,t)}(\delta; u_h) = \frac{\gamma\left(\Delta\eta(\tau, t; u_h), \frac{\delta}{\theta}\right)}{\Gamma\left(\Delta\eta(\tau, t; u_h)\right)},\ \ \delta \geq 0, \quad (5)$$

where:

$$\Delta\eta(\tau, t; u_h) = \int_\tau^t \eta'\left(y; u(y)\right) \cdot \mathrm{d}y = \int_\tau^t a\left(u(y)\right) \cdot b \cdot y^{b-1} \cdot \mathrm{d}y = a(u_h) \cdot (t^b - \tau^b).$$

Note that, as highlighted by the adopted notations, $\forall h, h = 1, \dots, k$ the pdfs $f_{W(\tau)}(\delta; u_0)$ and $f_{\Delta W(\tau,2\tau)}(\delta; u_h)$ and the cdfs $F_{W(\tau)}(\delta; u_0)$ and $F_{\Delta W(\tau,2\tau)}(\delta; u_h)$ depend on the usage rate.

Finally, it is not hard to verify that, under the same assumptions, for $t \leq \tau$, the conditional cdf of $W(t)$ given $W(\tau)$ can be expressed as:

$$F_{W(t)|W(\tau)}(w_t|w_\tau; u_0) = \mathbb{B}\left(\frac{w_t}{w_\tau}; \eta(t; u_0), \Delta\eta(t, \tau; u_0)\right) \quad (6)$$

where $\Delta\eta(t, \tau; u_0) = \eta(\tau; u_0) - \eta(t; u_0)$, and $\mathbb{B}(z; \alpha, \beta)$ is the regularized beta function:

$$\mathbb{B}(z; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \cdot \Gamma(\beta)} \cdot \int_0^z x^{\alpha-1} \cdot (1 - x)^{\beta-1} \cdot \mathrm{d}x.$$

4. The cost model

One of the main challenges of prescriptive maintenance is developing a cost model that is able to simultaneously take into account maintenance costs and operational costs, as well as the impact that prescriptive actions (e.g., changing the usage rate) have over it. In this paper, the cost model is developed considering the cost of a preventive replacement c_p, the cost of a corrective replacement $c_c\ \left(c_c \geq c_p\right)$, the inspection cost c_i, which is incurred only when an ad hoc inspection is done, and the logistic cost c_l, which is incurred each time an inspection or a replacement (even in the absence of an inspection) are performed.

The usage rate is assumed to impact the cost model through a reward term and a penalty term. Specifically, we assume that the reward earned by operating the unit at usage rate u can be computed as the product of a reward rate $r(u)$ and the operating time of the unit, while the penalty cost can be expressed as the product of a penalty rate $c_{pen}(u)$ and the downtime of the unit (i.e., the time elapsing from the potential failure of the unit until its eventual replacement). This penalty cost is sustained only in case of failure of the unit and is supposed to capture the effect of the reduced performances/additional costs resulting from operating the unit past its failure time.

Table 3 lists, for each possible scenario, the corresponding utility $U(w_\tau, X)$ as a function of the degradation level at the inspection time w_τ and of the lifetime of the unit X.

Table 3. Possible scenarios and corresponding utility.

Experimental scenario	Utility $U(w_\tau, X)$
$L_k < w_\tau \leq w_M$	$-2 \cdot c_l - c_i - c_p$ $+r(u_0) \cdot \tau$
$w_\tau > w_M$	$-2 \cdot c_l - c_i - c_c$ $+r(u_0) \cdot \tau$ $-c_{pen}(u_0) \cdot (\tau - X)$
$L_{k-1} < w_\tau \leq L_k$ and $w_{2\tau} \leq w_M$	$-2 \cdot c_l - c_i - c_p$ $+r(u_0) \cdot \tau + r(u_k) \cdot \tau$
$L_{k-1} < w_\tau \leq L_k$ and $w_{2\tau} > w_M$	$-2 \cdot c_l - c_i - c_c$ $+r(u_0) \cdot \tau + r(u_k) \cdot \tau$ $-c_{pen}(u_k) \cdot (2\tau - X)$
⋮	⋮
$L_1 < w_\tau \leq L_2$ and $w_{2\tau} \leq w_M$	$-2 \cdot c_l - c_i - c_p$ $+r(u_0) \cdot \tau + r(u_2) \cdot \tau$
$L_1 < w_\tau \leq L_2$ and $w_{2\tau} > w_M$	$-2 \cdot c_l - c_i - c_c$ $+r(u_0) \cdot \tau + r(u_2) \cdot \tau$ $-c_{pen}(u_2) \cdot (2\tau - X)$
$w_\tau \leq L_1$ and $w_{2\tau} \leq w_M$	$-2 \cdot c_l - c_i - c_p$ $+r(u_0) \cdot \tau + r(u_1) \cdot \tau$
$w_\tau \leq L_1$ and $w_{2\tau} > w_M$	$-2 \cdot c_l - c_i - c_c$ $+r(u_0) \cdot \tau + r(u_1) \cdot \tau$ $-c_{pen}(u_1) \cdot (2\tau - X)$

It is worth remarking that, given that failures are assumed to be not self-announcing, the lifetime X cannot be directly observed (not even conditionally to $W(\tau)=w_\tau$) and should always be regarded as a random variable. For this reason, in Tab. 3 it is always denoted with the uppercase letter. Despite the notation not highlighting it, $U(w_\tau, X)$ functionally depends on the decision parameter vector $\boldsymbol{\xi}$.

5. Formulation of the long-run average maintenance utility rate

The long-run average maintenance utility rate $U_\infty(\boldsymbol{\xi})$ is computed via the renewal-reward theorem (see Ross (1983)) as:

$$U_\infty(\boldsymbol{\xi}) = \frac{E\{U(W(\tau), X)\}}{E\{T(W(\tau))\}} \tag{7}$$

where expectations have to be taken with respect to both $W(\tau)$ and X.

The optimal value $U_\infty(\boldsymbol{\xi}^*)$ obtained when $\boldsymbol{\xi}=\boldsymbol{\xi}^*$ is denoted as U_∞^*.

The expected values in Eq. (7) are not available in closed form but can be computed via Eqs. (8)-(9). Here only a brief scratch of their derivation is given. More details can be found in Esposito et al. (2021).

$$\begin{aligned}
&E\{U(W(\tau), X)\} \\
&= \sum_{h=1}^{k} \int_{L_{h-1}}^{L_h} \int_{\tau}^{2\tau} U(w_\tau, x) \cdot f_{X|W(\tau)}(x|w_\tau; u_h) \\
&\qquad \times f_{W(\tau)}(w_\tau; u_0) \cdot \mathrm{d}x \cdot \mathrm{d}w_\tau \\
&+ \sum_{h=1}^{k} \int_{L_{h-1}}^{L_h} \int_{2\tau}^{\infty} U(w_\tau, x) \cdot f_{X|W(\tau)}(x|w_\tau; u_h) \\
&\qquad \times f_{W(\tau)}(w_\tau; u_0) \cdot \mathrm{d}x \cdot \mathrm{d}w_\tau \\
&+ \int_{L_k}^{w_M} \int_{\tau}^{\infty} U(w_\tau, x) \cdot f_{X|W(\tau)}(x|w_\tau; u_0) \\
&\qquad \times f_{W(\tau)}(w_\tau; u_0) \cdot \mathrm{d}x \cdot \mathrm{d}w_\tau \\
&+ \int_{w_M}^{\infty} \int_{0}^{\tau} U(w_\tau, x) \cdot f_{X|W(\tau)}(x|w_\tau; u_0) \\
&\qquad \times f_{W(\tau)}(w_\tau; u_0) \cdot \mathrm{d}x \cdot \mathrm{d}w_\tau \\
&= -c_l - c_i - c_p - c_l \cdot F_{W(\tau)}(L_k; u_0) + r(u_0) \cdot \tau \\
&\quad -[c_c - c_p + c_{pen}(u_0) \cdot \tau] \cdot [1 - F_{W(\tau)}(w_M; u_0)] \\
&\quad +\tau \cdot \sum_{h=1}^{k} [r(u_h) - c_{pen}(u_h)] \\
&\quad \times [F_{W(\tau)}(L_h; u_0) - F_{W(\tau)}(L_{h-1}; u_0)] \\
&\quad -(c_c - c_p) \cdot \sum_{h=1}^{k} \int_{L_{h-1}}^{L_h} [1 - F_{\Delta W(\tau, 2\tau)}(w_M - w_\tau; u_h)] \\
&\qquad \times f_{W(\tau)}(w_\tau; u_0) \cdot dw_\tau \\
&\quad + \sum_{h=1}^{k} \int_{L_{h-1}}^{L_h} \int_{\tau}^{2\tau} c_{pen}(u_h) \cdot F_{\Delta W(\tau, x)}(w_M - w_\tau; u_h) \\
&\qquad \times f_{W(\tau)}(w_\tau; u_0) \cdot dx \cdot dw_\tau \\
&\quad + c_{pen}(u_0) \cdot \int_{w_M}^{\infty} \int_{0}^{\tau} F_{W(x)|W(\tau)}(w_M | w_\tau; u_0) \\
&\qquad \times f_{W(\tau)}(w_\tau; u_0) \cdot dx \cdot dw_\tau
\end{aligned} \tag{8}$$

$$\begin{aligned}
&E\{T(W(\tau))\} = \\
&\quad \sum_{h=1}^{k} \int_{L_{h-1}}^{L_h} T(w_\tau) \cdot f_{W(\tau)}(w_\tau; u_0) \cdot \mathrm{d}w_\tau \\
&\quad + \int_{L_k}^{\infty} T(w_\tau) \cdot f_{W(\tau)}(w_\tau; u_0) \cdot \mathrm{d}w_\tau \\
&= 2 \cdot \tau \cdot \sum_{h=1}^{k} \int_{L_{h-1}}^{L_h} f_{W(\tau)}(w_\tau; u_0) \cdot \mathrm{d}w_\tau \\
&\quad + \tau \cdot \int_{L_k}^{\infty} f_{W(\tau)}(w_\tau; u_0) \cdot \mathrm{d}w_\tau \\
&= \tau \cdot [1 + F_{W(\tau)}(L_k; u_0)].
\end{aligned} \tag{9}$$

6. Example of application

We develop an example of application of the proposed policy to a real-world inspired case of a pipeline subjected to corrosion.

Frequently, pipelines are located in sites that are difficult to access (for example, buried underground and/or offshore) and hence only periodic inspections can be performed.

We assume that it is possible to control flow velocity according to convenience.

Indeed, as observed by Utanohara and Murase (2019) and Yoneda et al. (2016), flow velocity can influence the rate of corrosion. Specifically, we assume that flow velocity influences the scale parameter of the age function via a power-law function:

$$a(u) = a \cdot \left(\frac{u}{u_{max}}\right)^d, \tag{10}$$

whereas the reward rate $r(u)$ linearly depends on u:

$$r(u) = r \cdot \frac{u}{u_{max}}, \tag{11}$$

and the penalty cost rate is expressed as a constant fraction of $r(u)$:

$$c_{pen}(u) = c_{pen} \cdot r(u), \tag{12}$$

where in Eqs. (10)-(12) u_{max} is the maximum allowable flow velocity, and a, r, and c_{pen} are the corresponding maximum degradation rate, reward rate and penalty cost rate, respectively (i.e., $a = a(u_{max})$, $r = r(u_{max})$, and $c_{pen} = c_{pen}(u_{max})$).

Depending on the practical scenario under study, c_{pen} might be greater or smaller than 1 (that is, operating a failed unit may incur a penalty that is greater than the corresponding reward). In this example we assume that $c_{pen} = 0.8$, to investigate the case where the panalty is relevant but does not exceed the reward.

Tables 4 and 5 report the values of the parameters used to calibrate the degradation model and the cost model, respectively. These values have been set using as rough reference the ones found in Mahmoodian and Alani (2014) and Dey (2004).

The failure threshold w_M has been set to $35\ mm$. Table 6 reports the maximum and minimum allowable usage rates (denoted by u_{max} and u_{min}, respectively).

Table 4. Parameters used to calibrate the degradation process.

$a\ [years]$	b	$\theta\ [mm]$	d
0.24	1	2.35	2

Table 5. Parameters of the cost model.

c_p	c_c	c_i	c_l	c_{pen}	r
1	6	0.5	0.2	0.8	0.12

Table 6. Maximum and minimum values of the usage rate.

u_{max}	u_{min}
1	0

For the sake of comparison, let $P_0(k)$ be the proposed policy and P_1 be the special case of $P_0(k)$ where $k = 1$ and $u_0 = u_1$. Essentially, P_1 is a condition-based policy where the usage rate is set a priori to a fixed value that does not change during the maintenance cycle.

Fig. 1 depicts the optimal long-run average maintenance utility rate as a function of the inspection time τ under $P_0(k)$ (for $k = 1, 3,$ and 5) and under P_1.

This figure shows that, under the considered setup, adopting the more flexible policy $P_0(k)$ can provide noticeable improvements in the long-run average maintenance utility rate. To understand how these performances are achieved, it is necessary to investigate how $P_0(k)$ adaptively assigns the usage rate.

Fig. 2 depicts the optimal values of the usage rates u assigned by the considered policies in case $\tau = 35$, as a function of w_τ. Here, the preventive replacement threshold L_k coincides with the smallest value of w_τ where the assigned usage rate is 0.

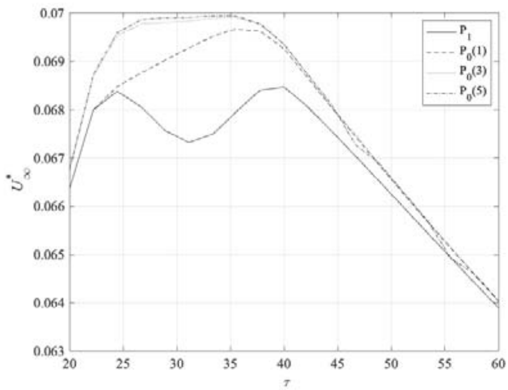

Figure 1. Optimal long-run average maintenance utility rate as a function of the inspection time τ under the considered policies.

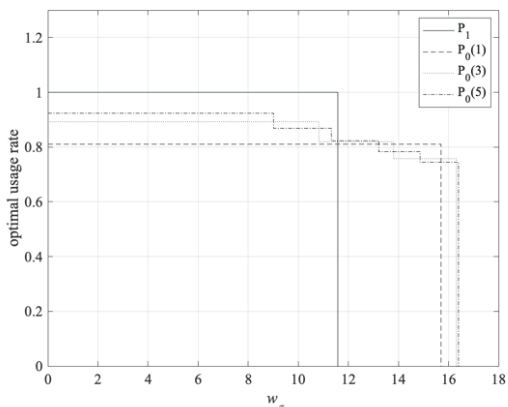

Figure 2. Optimal values of the usage rate in $(\tau, 2\tau)$ as a function of w_τ at $\tau = 35$ under $P_0(k)$ (with $k = 1, 3$ and 5) and P_1.

Fig. 2 shows that, under policy P_0, to a unit that is barely degraded at $\tau = 35$ it will be assigned a higher value of the usage rate, while to more degraded units it will be assigned a progressively lower usage rate.

Moreover, the same figure also shows that, due to the lack of flexibility, policy P_1 must be more conservative than $P_0(k)$. Indeed, under $P_0(k)$ the preventive replacement threshold is higher than under P_1, increasing the probability that the replacement is postponed, which in turn prolongs the operating life of the unit. As the number of classes k increases, this effect is further accentuated.

Let ${}_cU^*_\infty$, ${}_iU^*_\infty$, ${}_lU^*_\infty$, ${}_pU^*_\infty$, and ${}_{pen}U^*_\infty$ be the contribution to the optimal long-run average maintenance utility rate U^*_∞ of corrective, inspection, logistic, preventive, and penalty costs, respectively, and ${}_rU^*_\infty$ be the contribution to U^*_∞ of the reward term (obviously, it is ${}_cU^*_\infty + {}_pU^*_\infty + {}_lU^*_\infty + {}_iU^*_\infty + {}_{pen}U^*_\infty + {}_rU^*_\infty = U^*_\infty$). The bar chart in Fig. 3 shows (in black) the values of these contributions under policy P_1 and (in grey) under policy $P_0(5)$ (i.e., $P_0(k)$ with $k = 5$). The same values are also reported in Tab. 7, together with the total cost U^*_∞.

Fig. 3 and Tab. 7 show that the reward earned under P_1 is greater than the one earned under $P_0(5)$ (i.e., the contribution of the reward term ${}_rU^*_\infty$ is higher under P_1 than under $P_0(5)$). However, this is compensated by the contributions of all the other (negative) cost factors, which are all smaller than under $P_0(5)$.

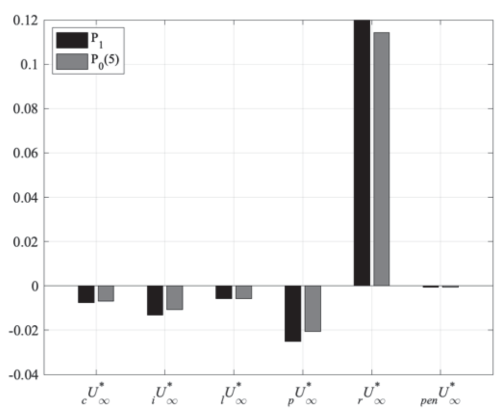

Figure 3. Values of ${}_cU^*_\infty$, ${}_iU^*_\infty$, ${}_lU^*_\infty$, ${}_pU^*_\infty$, ${}_rU^*_\infty$, and ${}_{pen}U^*_\infty$ obtained under P_1 (in black) and under $P_0(5)$ (in grey).

These results show that, by adaptively assigning a usage rate tailored to the actual degradation level of the unit, the policy $P_0(5)$ is able to prolong the useful life of the unit while carefully managing the risk of failure.

Table 7. Values of ${}_cU^*_\infty$, ${}_iU^*_\infty$, ${}_lU^*_\infty$, ${}_pU^*_\infty$, ${}_rU^*_\infty$, ${}_{pen}U^*_\infty$, and U^*_∞ obtained under P_1 and under $P_0(5)$.

	$P_0(5)$	P_1
${}_cU^*_\infty$	−0.0068	−0.0076
${}_iU^*_\infty$	−0.0108	−0.0132
${}_lU^*_\infty$	−0.0057	−0.0057
${}_pU^*_\infty$	−0.0205	−0.0251
${}_rU^*_\infty$	0.1144	0.12
${}_{pen}U^*_\infty$	$-5.49 \cdot 10^{-4}$	$-6.64 \cdot 10^{-4}$
U^*_∞	0.0699	0.0668

These results show that, by adaptively assigning a usage rate tailored to the actual degradation level of the unit, the policy $P_0(5)$ is able to prolong the useful life of the unit while carefully managing the risk of failure.

7. Conclusions

In this paper, we have proposed an adaptive prescriptive maintenance policy for a gamma degrading unit that generalizes the one proposed in Esposito et al. (2022).

Inspired by modern prescriptive maintenance concepts, the policy includes among the possible actions, besides the classical inspection, corrective replacement, and preventive replacement, also the possibility of changing the usage rate of the considered unit during its operating life.

Specifically, the policy consists in performing a single inspection at a fixed time and, based on the outcome of this inspection, deciding whether to replace the unit immediately or postpone its replacement to a later time. In this latter case, the usage rate of the unit for the remainder of its operating life can be set to a new value, if deemed economically convenient.

The main novelty with respect to Esposito et al. (2022) is that here the new value of the usage rate is adaptively assigned based on the measured degradation level at the inspection time.

This policy is particularly suitable for practical applications where, due to external logistic/economic reasons, maintenance actions can be performed only periodically and at prearranged times.

The rationale behind this policy is that, when maintenance times cannot be changed, the additional degree of freedom provided by the possibility of changing the usage rate can help in finding a better tradeoff between preventive replacement, corrective replacement, inspection, and operational costs.

The impact of these actions both on the degradation process and on the cost model has been taken into account. The degradation process of the unit is described by a gamma process. Units are considered failed based on a failure threshold model. The optimal maintenance policy has been defined by using as a performance measure the long-run average maintenance utility rate.

Finally, an example of application of the suggested strategy to a real-world inspired case of a corroding pipeline has been developed. Obtained results show that, by adaptively assigning the usage rate based on the degradation level measured at the inspection time, the proposed policy is able to prolong and better exploit the operational life of the unit while simultaneously managing the risk of failure and ultimately leading to a greater overall utility compared to a more classical approach.

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