Proceedings of the 33rd European Safety and Reliability Conference (ESREL 2023)
Edited by Mário P. Brito, Terje Aven, Piero Baraldi, Marko Čepin and Enrico Zio

doi: 10.3850/978-981-18-8071-1_P318-cd

Integrated Planning of Usage-Based Maintenance and Load Sharing Under Resource Dependence

Halit Metehan Dilaver

Department of Industrial Engineering and Innovation Sciences, Eindhoven University of Technology, The Netherlands. E-mail: h.m.dilaver@tue.nl

Alp Akcay

Department of Industrial Engineering and Innovation Sciences, Eindhoven University of Technology, The Netherlands. E-mail: a.e.akcay@tue.nl

Geert-Jan van Houtum

Department of Industrial Engineering and Innovation Sciences, Eindhoven University of Technology, The Netherlands. E-mail: g.j.v.houtum@tue.nl

In many systems, functionally interchangeable units are used together to meet a common demand or production target. Such examples include parallel machines in production facilities, engines of a vessel, and fleets of ships, airplanes, or trucks. These units typically receive large-scale maintenance dependent on their usage (such as overhauls) and therefore, the timing of their maintenance is directly affected by the policy that determines how the total demand is allocated to the units. We assume that there is a limit on how many units can get maintenance simultaneously because of the limited resources that are involved (e.g., a dry-dock, hangar, or specialized workforce) and/or because the demand needs to be met at all times. In this study, the problem of integrated planning of usage-based maintenance and load sharing (i.e., the allocation of total demand to different units) for multi-unit systems is mathematically analyzed. Also, a mathematical model is built to minimize the total maintenance costs during the finite lifetime of the units (which is generally 10 to 40 years). An asymptotically near-optimal policy is proposed, and its performance is compared with the performance of the optimal policy.

Keywords: Integrated Planning, Usage-Based Maintenance, Load Sharing, Resource Dependence.

1. Introduction

The joint planning of operations and maintenance for a fleet of units is a complex process. Every maintenance decision reduces the operational capacity of the fleet, while the policies for the allocation of the workload affect the usage amount of the units, and therefore, their maintenance needs. Additionally, for systems with a limited number of resources for maintenance operations, the number of maintenance operations that can be done simultaneously is also limited. That is why it is essential to consider the link between usage and maintenance.

Recently, there has been a noticeable increase in researchers' interest in joint planning of operations and maintenance. Motivated by applications in various sectors, many optimization models have been built for multi-unit systems. For instance, Olde Keizer et al. (2018) study a 1-out-of-n system inspired by a gas company. In that problem, the continuity of production is critical to meeting the gas demands of companies and households. The authors build a Markov decision process model, which considers an economic dependence on the units and workload-dependent failure rates to minimize average long-run cost. Basciftci et al. (2020) address the problem of scheduling maintenance and operations for a fleet of generators, where the degradation of generators is directly affected by how they are used. The authors aim to create a solution that considers the limitation of full demand satisfaction, which may require demand curtailment in the event of unexpected production losses. In doing so, they develop a planning framework that maximizes production while minimizing costs and including the generators' degradation due to their usage.

More recently, Uit Het Broek et al. (2020) introduce the concept of *condition-based production*. This novel approach aims to control the level of degradation and maintenance time by dynamically adjusting the production rate of a single-unit system. They build a model that maximizes the total output until the time of pre-scheduled maintenance. Uit Het Broek et al. (2021) extend the work of Uit Het Broek et al. (2020) to two-unit systems. They assume a fixed minimum production target per period and an economic dependency on the maintenance operations of the units. These two papers show that condition-based production is effective in reducing costs for both single and multi-unit systems.

Although there are studies that analyze the operations and maintenance planning problems analytically for both single-unit systems (e.g., Uit Het Broek et al. 2020; Drent et al. 2023) and multi-unit systems (e.g., Ashizawa and Lu 2022 under economic dependence, studies that consider the *resource dependence* are hardly available. This dependency type arises when there is a commonly used resource (e.g., tools, labors, facilities) for maintenance operations of a multi-unit system. Dilaver et al. (2023) study the problem of integrated planning of asset-use and dry-docking for maritime fleets under resource dependence. However, no study has yet been conducted to analyze this problem analytically.

In this paper, the problem of integrated planning of usage-based maintenance and load sharing (i.e., the allocation of total demand to different units) under resource dependence is investigated. The focus of this paper is on multi-unit systems such as parallel machines in production facilities, engines of a vessel, and fleets of ships, airplanes, or trucks, where the units are functionally interchangeable to meet a common demand or production target. The main objective of this study is to introduce a practically relevant policy called *staircase policy* that can optimize the maintenance schedule and load sharing among units to ensure that the demand is fully satisfied at every time period while minimizing the total cost of maintenance over a finite horizon. Also, we evaluate the performance of the staircase policy both analytically and numerically, and show that it is asymptotically near-optimal. The maintenance operations considered in this paper are usage-based maintenance operations called *overhaul*. After each overhaul, the units become as good as new. The other maintenance types are left out of scope. The words maintenance and overhaul will be used interchangeably in the rest of the paper.

The remainder of the paper is organized as follows. Section 2 describes the problem formulation. In Section 3, the feasibility condition of a problem instance is introduced, and the structural properties of the optimal policy are discussed. Additionally, the staircase policy and its asymptotic near-optimality are presented. Section 4 describes how to obtain an optimal integrated planning solution. Section 5 presents the results of the numerical study where the lower-bound, asymptotically optimal policy, and optimal policy are compared. Finally, Section 6 concludes the paper by highlighting our contributions.

2. Problem Formulation

We consider a fleet consisting of identical and functionally exchangeable units. Those units are used to satisfy a common demand at a single location over a finite planning horizon. The horizon is divided into T periods of equal length, and each period has a length of δ days. The periods are numbered as $1, 2, \ldots, T$ and the length of the whole horizon is equal to δT days. There is a constant operational demand per period, denoted as D in terms of operating hours. This constant amount of demand has to be satisfied by a fleet of N units, which each have a maximum capacity C per unit per period in terms of operating hours. It is assumed that throughout the time horizon the number of the units N is fixed. There is a single maintenance facility available for overhauls. The model begins with a whole new fleet of units, each requiring an overhaul after at most K operating hours. The duration of an overhaul is one period, and the cost for each overhaul, which includes the related costs for the unit to be overhauled, e.g., labor cost, facility cost, and spare part cost, is assumed to be fixed and denoted by C^o. It is required that the demand is fully satisfied at all time periods. The objective is to minimize the total overhaul cost. We make the following assumption.

Assumption 2.1. $(A1)$ $D \leq (N-1)C$, $(A2)$ $C \leq K$, *and* $(A3)$ $C \leq D$.

$(A1)$ is necessary for the feasibility of the problem since the demand has to be satisfied even if one of the units is under maintenance. Assumptions $(A2)$ and $(A3)$ can be assumed without loss of generality as a capacity C greater than the limit K or the demand per period D cannot be fully utilized.

3. Analysis

We first present a sufficient condition for the feasibility of a problem instance, and a lower-bound on the optimal number of overhauls in Section 3.1. Then, we formally introduce the staircase policy and its analysis in Section 3.2.

3.1. *Preliminary Analysis*

Let $(y_{1,t}, y_{2,t}, ..., y_{N,t})$ denote the system's state at the end of period t, where $y_{i,t}$ denotes the cumulative usage level of unit i since its last overhaul in terms of operating hours. We assume without loss of generality that the units are ordered in descending order based on their cumulative usage levels. That is for any state $(y_{1,t}, y_{2,t}, ..., y_{N,t})$ at the end of period $t \in \{1, \ldots, T\}$, it holds that $y_{1,t} \geq y_{2,t} \geq ... \geq y_{N,t}$.

Definition 3.1. The state $(y_{1,t}, y_{2,t}, ..., y_{N,t})$ is feasible if and only if there is a policy for the periods $t+1, t+2, \ldots, T$, which guarantees full demand satisfaction every time period and follows the rules and restrictions of overhaul planning.

Proposition 3.1. *The initial state, i.e,* $(0, 0, ..., 0)$ *at* $t = 0$*, is feasible if* $K \geq D$.

Proof. Let policy P be a policy where the most used unit (i.e., in case of equality, it can be chosen randomly among the most used units) is to be overhauled in every period while the demand is to be equally shared among the other units. Since $D \leq (N-1)C$, then the state transition of the system will be as follows under policy P:
The system is initiated at time 0 at state $(0, 0, ..., 0)$. In the first period, unit 1, which corresponds to $y_{1,0}$ at the initial state, will be overhauled, and the demand will be equally shared among the other units. Then, the state will become $(\frac{D}{(N-1)}, \frac{D}{(N-1)}, ..., \frac{D}{(N-1)}, 0)$. In period 2, after the unit corresponding to $y_{1,1}$ is overhauled, and the demand is equally shared, the state will become $(\frac{2D}{(N-1)}, ..., \frac{2D}{(N-1)}, \frac{D}{(N-1)}, 0)$. By applying the same policy for $(N-1)$ periods, the system state will be $(\frac{(N-1)D}{(N-1)}, \frac{(N-2)D}{(N-1)}, ..., \frac{D}{(N-1)}, 0)$. As the most used unit will be overhauled in every period and $\frac{D}{(N-1)}$ amount of workload will be assigned to the rest of the units, once the system goes into state $(\frac{(N-1)D}{(N-1)}, \frac{(N-2)D}{(N-1)}, ..., \frac{D}{(N-1)}, 0)$, it will stay there for the rest of the horizon. Note that the highest level reached by the cumulative usage level of a unit is the level of D. Hence, state $(0, 0, ..., 0)$ is feasible if $K \geq D$. □

Based on Proposition 3.1 and Assumption 2.1, the following assumption is made for the remainder of the paper.

Assumption 3.1. $C \leq D \leq K$

Theorem 3.1. *The lower-bound on the minimum number of overhauls is equal to* $(\lceil \frac{DT}{K} \rceil - N)^+$*. Hence, the corresponding overhaul cost is equal to* $(\lceil \frac{DT}{K} \rceil - N)^+ C^o$.

Proof. Since there are T periods and D is the demand per period, the total amount of demand to be satisfied by the fleet is equal to TD. Note that, as stated in Section 2, at the beginning of the planning horizon, every unit can be used at most K operating hours before its next overhaul. This means that the fleet of N units can be used at most NK operating hours until the next overhaul. After each overhaul, no matter how many operating hours are left, the remaining operating hours is reset to K. Hence, for the remaining total demand after NK, the minimum number of overhauls is equal to $\frac{DT-NK}{K}$. As the number of overhauls has to be a non-negative integer, the lower-bound is $(\lceil \frac{DT}{K} \rceil - N)^+$ and the total cost of overhauls is $(\lceil \frac{DT}{K} \rceil - N)^+ C^o$. □

A common policy that can be used as a benchmark for integrated-planning policies is *Equal-Sharing Policy*, where the demand is always equally shared among functional units. However, for systems with resource dependency, the equal sharing policy might not be feasible as it leads to a state where all units need to be maintained at the same time.

Theorem 3.2. *If $\lceil \frac{TD}{K} \rceil \leq N$, then the Equal-Sharing Policy and doing no overhauls is optimal, and the optimal objective value is equal to zero.*

Proof. Let $\lceil \frac{TD}{K} \rceil \leq N$. Then, by the property of the ceiling function, $\frac{TD}{K} \leq \lceil \frac{TD}{K} \rceil \leq N$. Therefore, $\frac{TD}{N} = T\frac{D}{N} \leq K$. Therefore, if the demand is equally shared among all the units for T periods, (i.e., the workload will be equal to $\frac{D}{N}$ per unit per period) then the cumulative usage levels of the units will be less then or equal to the limit K. Since $\frac{D}{N} \leq \frac{D}{N-1}$ and by Assumption 2.1-$(A1)$, $D \leq (N-1)C$, the Equal-Sharing Policy is feasible and leads to zero overhauls for any value of C. Hence, it is optimal. □

Since it is proven that the optimal policy and the optimal solution are obvious when $\lceil \frac{TD}{K} \rceil \leq N$, the following assumption is made.

Assumption 3.2. $\lceil \frac{TD}{K} \rceil > N$

3.2. *Staircase Policy*

The staircase policy aims to achieve a staircase-looking state, where there is an equal difference Δ between the cumulative usage levels of units and the equal-sharing policy is feasible afterward. In other words, the objective is to establish a feasible approach wherein the workload is shared equally among functional units (not under maintenance). In Figure 1, an example of the staircase state for N units is given, where the vertical axis denotes the cumulative usage level of the units and the horizontal axis denotes the unit index.

In this study, we propose to use $\Delta = D/(N-1)$ because then the other units can fully satisfy the demand when the first unit goes into overhaul. In order to achieve the staircase state where $\Delta = D/(N-1)$, we propose a starting policy that

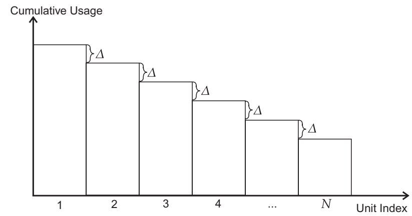

Fig. 1. An example state with an equal staircase difference Δ between the cumulative usage of units.

uses $N-1$ units equally, i.e., $\frac{D}{N-1}$, for every time period until a staircase state is achieved. We suppose that the unit indexed with 1 will be used the most, and the unit indexed with N will be used the least. Consider that for τ_1 periods, unit 1 is used for every period, while unit 2 is used every period but one, unit 3 is used every period but two, etc., and unit N is used for every period but $N-1$ periods. Since the total number of $N(N-1)/2$ (i.e., $0+1+\ldots+(N-1)$) free turns is needed, τ_1 is equal to $\frac{N(N-1)}{2}$ periods. At the end of the τ_1 periods, the cumulative usage of unit 1 is $\tau_1 \frac{D}{(N-1)} = \frac{DN}{2}$. By this policy, the staircase state, where $y_{1,\tau_1} = \frac{DN}{2}$ and $\Delta = \frac{D}{N-1}$ can be achieved. Note that the order of free turns of the units until the staircase state is achieved does not matter. However, this starting policy is feasible only if $\frac{DN}{2} \leq K$. See Figure 2. In real-life applications, the overhaul limit K is high enough that the units will be overhauled every few years on average. Therefore, the following assumption can be made.

Assumption 3.3. $\frac{DN}{2} \leq K$

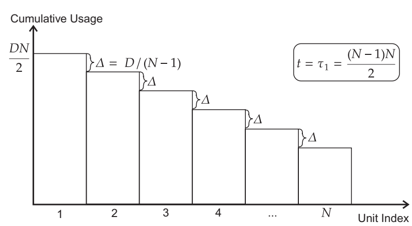

Fig. 2. The staircase state is achieved at the end of period τ_1.

After the staircase state is reached, the equal sharing policy can be used until the time $t = \tau_1 + \tau_2$ at which $K' = \frac{DN}{2} + i\frac{D}{N}$ for some non-negative, integer-valued i and $K - K' < \frac{D}{N}$. Combining these two conditions leads to the following closed-form expressions for τ_2 and K': $\tau_2 = \lfloor \frac{KN}{D} - \frac{N^2}{2} \rfloor$ and $y_{1,\tau_1+\tau_2} = K' = \lfloor \frac{KN}{D} - \frac{N^2}{2} \rfloor \frac{D}{N} + \frac{DN}{2}$. See Figure 3. Note that if $T \leq \tau_1 + \tau_2$ then the staircase state of Figure 3 may not be reached before the end of the horizon. However, since there is no overhaul in the first $\tau_1 + \tau_2$ periods, then the staircase policy leads to zero overhauls (maintenance cost) which is the natural minimum number of overhauls and hence optimal.

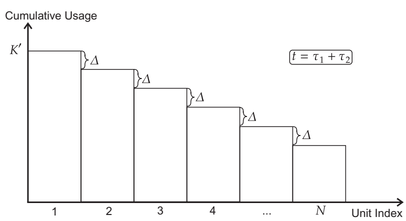

Fig. 3. The staircase state where $y_{1,\tau_1+\tau_2} = K'$ at the end of period $\tau_1 + \tau_2$.

Next, since unit 1 is the most used unit at cumulative usage level K', in period $\tau_1 + \tau_2 + 1$, unit 1 goes into overhaul and the load is equally shared among the other units, i.e., each unit gets a load of $\frac{D}{(N-1)} = \Delta$. Then, in period $\tau_1 + \tau_2 + 2$, the next most used unit goes into overhaul, and so on. This allows, after N periods, the equal staircase difference to be achieved and the equal-sharing policy to be feasible again. As the workload assigned to the unit 1 will be equal to Δ for the next $N - 1$ periods after its maintenance, it holds that $y_{1,\tau_1+\tau_2+N} = (N-1)\Delta = D$. See Figure 4.

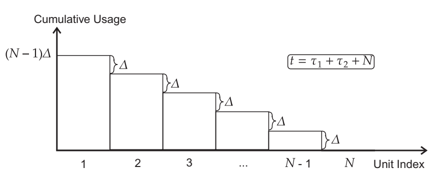

Fig. 4. The staircase state where $y_{1,\tau_1+\tau_2+N} = (N-1)\Delta$ at the end of period $\tau_1 + \tau_2 + N$.

As the equal sharing policy is feasible again, it can be followed until the time $\tau_1 + \tau_2 + N + \tau_3$ at which $K'' = (N-1)\Delta + j\frac{D}{N}$ for some non-negative, integer-valued j and $K - K'' < \frac{D}{N}$. The combination of these two conditions leads to the following expressions of τ_3 and K'': $\tau_3 = \lfloor \frac{KN}{D} - N \rfloor$ and $y_{1,\tau_1+\tau_2+N+\tau_3} = K'' = D + \lfloor \frac{KN}{D} - N \rfloor \frac{D}{N}$. See Figure 5.

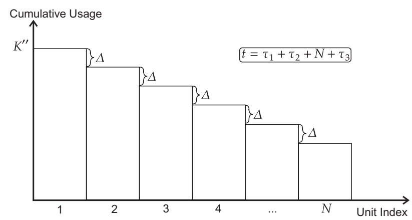

Fig. 5. The staircase state where $y_{1,\tau_1+\tau_2+N+\tau_3} = K''$ at the end of period $\tau_1 + \tau_2 + N + \tau_3$.

Whenever the level K'' is reached, the staircase policy orders maintenance for the next N period, as in the case $y_{1,\tau_1+\tau_2} = K'$. Similarly, for N periods, the demand will be equally shared among the units except for the one under maintenance, i.e., $\frac{D}{(N-1)} = \Delta$. After that, the equal staircase difference will be achieved, and the equal-sharing policy will be feasible again. As the workload assigned to unit 1 will be equal to Δ for the next $N - 1$ periods after its maintenance $y_{1,\tau_1+\tau_2+N+\tau_3+N} = (N-1)\Delta = D$. See Figure 4.

After reaching the state shown in Figure 3 at time $t = \tau_1 + \tau_2$, the same overhaul scheme will be received for every cycle of $N + \tau_3$ periods and the number of overhauls will be equal to N for the cycle of $N + \tau_3$ periods. At the end of every cycle, the state of the system will become as in Figure 5. Note that the staircase policy is feasible for every capacity limit and, therefore, easy to adapt in real-life systems.

Notice that for the first $\tau_1 + \tau_2$ periods, there is no overhaul if $K \geq \frac{DN}{2}$. After that, the remaining planning horizon can be divided into cycles of $N + \tau_3$ periods. For every completed cycle of $N + \tau_3$ periods, the number of overhauls is equal to N. The number of completed cycles is

$\lfloor \frac{T-(\tau_1+\tau_2)}{N+\tau_3} \rfloor$. If there is an uncompleted cycle at the end of the horizon, then the length of this cycle is $T-(\tau_1+\tau_2)-\lfloor \frac{T-(\tau_1+\tau_2)}{N+\tau_3} \rfloor (N+\tau_3)$. Therefore, since the overhauls are applied in the first N periods of each cycle, the number of overhauls for that uncompleted cycle is $min\{N, T-(\tau_1+\tau_2)-\lfloor \frac{T-(\tau_1+\tau_2)}{N+\tau_3} \rfloor (N+\tau_3)\}$. Then, the number of overhauls to be made as the result of the staircase policy, which is denoted by $f(N,K,D,C,T)$, can be calculated with the following closed-form mathematical expression:

$$f(N,K,D,C,T) = \lfloor \frac{T'}{N+\tau_3} \rfloor N + min\{N, T' - \lfloor \frac{T'}{N+\tau_3} \rfloor (N+\tau_3)\},$$

where $T' = T-(\tau_1+\tau_2)$.

Theorem 3.3. *The staircase policy is an asymptotically $\alpha-$optimal policy (i.e., the ratio of the cost of the staircase policy to the optimal cost is not greater than α), where $\alpha = \frac{N}{\lfloor \frac{KN}{D} \rfloor} \frac{K}{D}$, when T goes to infinity. Additionally, if $K'' = K$, then the staircase policy is asymptotically optimal when T goes to infinity.*

Proof. Let the number of overhauls under the staircase policy be $f(N,K,D,C,T)$, which is a function of the parameters N, K, D, C, and T. Let LB denote the lower-bound presented in Section 3. Then,

$$\begin{aligned} f(N,K,D,C,T) &= \lfloor \frac{T'}{N+\tau_3} \rfloor N \\ &+ min\{N, T' - \lfloor \frac{T'}{N+\tau_3} \rfloor (N+\tau_3)\} \\ &\leq \frac{T'}{N+\tau_3} N + N, \end{aligned}$$

where $T' = T-(\tau_1+\tau_2)$. Recall that; $LB = (\lceil \frac{TD}{K} \rceil - N)^+ \geq \frac{TD}{K} - N$ Hence,

$$\lim_{T\to\infty} \frac{f(N,K,D,C,T)}{LB} \leq \lim_{T\to\infty} \frac{\frac{T-\tau_1-\tau_2}{N+\tau_3} N + N}{\frac{TD}{K} - N}$$
$$= \frac{N}{N+\tau_3} \frac{K}{D} = \frac{N}{N + \lfloor \frac{KN}{D} - N \rfloor} \frac{K}{D} = \frac{N}{\lfloor \frac{KN}{D} \rfloor} \frac{K}{D}$$

This means that the ratio of the result of the staircase policy to the lower-bound over the optimal is limited by $\alpha = \frac{N}{\lfloor \frac{KN}{D} \rfloor} \frac{K}{D}$. This concludes the staircase policy is an asymptotically $\alpha-$optimal policy when T goes to infinity.

For $\frac{KN}{D} \in \mathbb{N}$,

$$\begin{aligned} 1 \leq \lim_{x\to\infty} \frac{f(N,K,D,C,T)}{LB} &\leq \frac{N}{\lfloor \frac{KN}{D} \rfloor} \frac{K}{D} \\ &= \frac{N}{\frac{KN}{D}} \frac{K}{D} = \frac{D}{K} \frac{K}{D} = 1 \end{aligned}$$

Therefore, when T goes to infinity, the result of the staircase policy is equal to the lower-bound and to the optimal. □

4. Optimal Solution

In Section 3.2, the staircase policy is presented, and it is proven to be asymptotically $\alpha-$optimal. However, as stated in Section 2, the planning horizon is assumed to be finite for this study as the lifetime of the units is limited. In order to evaluate the performance of the staircase policy over a finite planning horizon, a Mixed Integer Linear Programming (MILP) model is built to find the optimal policy. The built model is a simplified version of the MILP model presented by Dilaver et al. (2023). Note that, in their model, they consider both usage-based and calendar-time-based maintenance operations. Since calendar-time-based maintenance operations are not within the scope of this study, the related decision variables, cost items, and constraints are excluded. Additionally, the number of locations is fixed as one and the demand is equal in all periods.

5. Numerical Results

In this section, we present the results of the numerical study to evaluate the performance of the staircase policy over a set of finite-horizon problem instances. For each problem instance, every period is equal to 1 week, and the overhaul limit K is equal to 10000 operating hours. For performance evaluation, 5 levels of the length of the time horizon T, i.e., 520 (10 years), 1040 (20 years), 1560 (30 years), 2080 (40 years) and 2600 (50 years), 3 levels of N (i.e., 3, 4, and 5), 3 levels of unit capacity per period C, i.e., 112 (16 operating hours per day), 140 (20 operating hours per day), and 168 (24 operating hours per day), and 3 levels for the demand per period, i.e., $(N-1)C$,

$(N-1.5)C$, and $(N-2)C$, are used. For every problem instance, a fixed level of K is used. For a total of 135 problem examples, the lower-bounds, the optimal results obtained by solving the MILP model and the results of the staircase policy were compared. Due to the limited number of pages of the article, 81 of the instances, which correspond to the number of periods 520, 1560 and 2600, are presented in Table 1. For each problem instance i, the corresponding number of units N, the capacity per unit per period C, i.e., in terms of operating hours, the amount of demand per period D, i.e., in terms of operating hours, and the number of periods T are specified. Besides these parameters, the lower-bound on the number of overhauls, i.e., LB, the number of overhauls obtained by the MILP model, i.e., Opt, and the number of overhauls obtained by the staircase policy, i.e., SCP, are presented in Table 1. Also, for each instance the gap between the Opt and the SCP is given in the $Gap\%$ and the $|Gap|$ columns (i.e., $Gap\% = (SCP - Opt)/Opt$ and $|Gap| = |SCP - Opt|$).

In Table 1, the staircase policy led to the optimal result in 30 of 81 instances (49 of 135). Also, the lower bound is equal to the optimal result in 60 of the 81 instances (103 of 135).

For all the 135 instances, the biggest difference between the staircase policy and the optimal policy is 25.00% in terms of $Gap\%$, while it is 4 in terms of $|Gap|$. The largest relative difference between the optimal and the lower bound is quite low at 5.80%. In this respect, it can be said that the lower bound is quite tight. In order to see the performance of the staircase policy with respect to the length of the time horizon and the number of units, the average gap values are calculated and presented in Table 2. For the subset of instances with the same values for T and N, see Table 1, where we present the values for $Gap\%$ and $|Gap|$. In Table 2, $Gap\%$ and the $|Gap|$ are presented for all 5 levels for T (rows) and all 3 levels for N (columns).

Table 2 shows that the average relative gap between the staircase policy and the optimal policy is less than 10% for almost all combinations of T and N. While the gap tends to increase as the number of units increases, it tends to decrease for an increasing number of periods. Since the service life is typically 20 years or more for the fleets of vessels and aircraft, it is a practically important insight that the average gap is always less than 7% for all 3 levels of the number of units for the

Table 1. Comparison of the numerical results of the optimal policy and the staircase policy.

i	N	C	D	T	LB	Opt	SCP	$Gap\%$	$\|Gap\|$
1	3	112	224	520	9	9	9	0.00%	0
2	3	112	224	1560	32	32	33	3.13%	1
3	3	112	224	2600	56	57	57	0.00%	0
4	3	112	168	520	6	6	6	0.00%	0
5	3	112	168	1560	24	24	24	0.00%	0
6	3	112	168	2600	41	41	42	2.44%	1
7	3	112	112	520	3	3	3	0.00%	0
8	3	112	112	1560	15	15	15	0.00%	0
9	3	112	112	2600	27	27	27	0.00%	0
10	3	140	280	520	12	12	12	0.00%	0
11	3	140	280	1560	41	42	42	0.00%	0
12	3	140	280	2600	70	71	72	1.41%	1
13	3	140	210	520	8	8	9	13.00%	1
14	3	140	210	1560	30	30	30	0.00%	0
15	3	140	210	2600	52	52	54	3.85%	2
16	3	140	140	520	5	5	6	20.00%	1
17	3	140	140	1560	19	19	21	10.53%	2
18	3	140	140	2600	34	34	36	5.88%	2
19	3	168	336	520	15	15	15	0.00%	0
20	3	168	336	1560	50	50	51	2.00%	1
21	3	168	336	2600	85	87	87	0.00%	0
22	3	168	252	520	11	11	12	9.00%	1
23	3	168	252	1560	37	37	39	5.41%	2
24	3	168	252	2600	63	63	63	0.00%	0
25	3	168	168	520	6	6	6	0.00%	0
26	3	168	168	1560	24	24	24	0.00%	0
27	3	168	168	2600	41	41	42	2.44%	1
28	4	112	336	520	14	14	16	14.00%	2
29	4	112	336	1560	49	49	52	6.12%	3
30	4	112	336	2600	84	84	84	0.00%	0
31	4	112	280	520	11	11	12	9.00%	1
32	4	112	280	1560	40	40	40	0.00%	0
33	4	112	280	2600	69	69	72	4.35%	3
34	4	112	224	520	8	8	8	0.00%	0
35	4	112	224	1560	31	31	32	3.23%	1
36	4	112	224	2600	55	55	56	1.82%	1
37	4	140	420	520	18	18	20	11.00%	2
38	4	140	420	1560	62	63	64	1.59%	1
39	4	140	420	2600	106	108	108	0.00%	0
40	4	140	350	520	15	15	16	7.00%	1
41	4	140	350	1560	51	51	52	1.96%	1
42	4	140	350	2600	87	88	88	0.00%	0
43	4	140	280	520	11	11	12	9.00%	1
44	4	140	280	1560	40	40	40	0.00%	0
45	4	140	280	2600	69	69	72	4.35%	3
46	4	168	504	520	23	23	24	4.00%	1
47	4	168	504	1560	75	76	76	0.00%	0
48	4	168	504	2600	128	128	128	0.00%	0
49	4	168	420	520	18	18	20	11.00%	2
50	4	168	420	1560	62	62	64	3.23%	2
51	4	168	420	2600	106	107	108	0.93%	1
52	4	168	336	520	14	14	16	14.00%	2
53	4	168	336	1560	49	49	52	6.12%	3
54	4	168	336	2600	84	84	84	0.00%	0
55	5	112	448	520	19	19	20	5.00%	1
56	5	112	448	1560	65	69	70	1.45%	1
57	5	112	448	2600	112	114	115	0.88%	1
58	5	112	392	520	16	16	20	25.00%	4
59	5	112	392	1560	57	57	60	5.26%	3
60	5	112	392	2600	97	99	100	1.01%	1
61	5	112	336	520	13	13	15	15.00%	2
62	5	112	336	1560	48	48	50	4.17%	2
63	5	112	336	2600	83	83	85	2.41%	2
64	5	140	560	520	25	25	25	0.00%	0
65	5	140	560	1560	83	84	85	1.19%	1
66	5	140	560	2600	141	145	145	0.00%	0
67	5	140	490	520	21	21	25	19.00%	4
68	5	140	490	1560	72	73	75	2.74%	2
69	5	140	490	2600	123	125	125	0.00%	0
70	5	140	420	520	17	17	20	18.00%	3
71	5	140	420	1560	61	61	65	6.56%	4
72	5	140	420	2600	105	105	105	0.00%	0

i	N	C	D	T	LB	Opt	SCP	$Gap\%$	$\lvert Gap \rvert$
73	5	168	672	520	30	31	35	13.00%	4
74	5	168	672	1560	100	102	105	2.94%	3
75	5	168	672	2600	170	175	175	0.00%	0
76	5	168	588	520	26	26	30	15.00%	4
77	5	168	588	1560	87	88	90	2.27%	2
78	5	168	588	2600	148	150	150	0.00%	0
79	5	168	504	520	22	22	25	14.00%	3
80	5	168	504	1560	74	74	75	1.35%	1
81	5	168	504	2600	127	127	130	2.36%	3

Table 2. Average of gaps between the optimal policy and the staircase policy.

N	3		4		5	
T	$\lvert Gap \rvert$	$Gap\%$	$\lvert Gap \rvert$	$Gap\%$	$\lvert Gap \rvert$	$Gap\%$
520	0.333	4.62%	1.333	8.89%	2.778	13.81%
1040	0.333	1.52%	1.333	4.12%	2.889	6.78%
1560	0.667	2.34%	1.222	2.47%	2.111	3.10%
2080	0.778	1.80%	0.889	1.48%	1.667	2.06%
2600	0.778	1.78%	0.889	1.27%	0.778	0.74%

planning horizons of 20 years and longer.

6. Conclusion

In this study, a problem of integrated planning for usage-based maintenance and load-sharing under resource dependence is introduced. The problem considers making the large-scale maintenance decisions of units in a fleet and the load sharing decisions of the units simultaneously, with the aim of minimizing the total cost of maintenance while ensuring demand satisfaction. For this purpose, a practically applicable policy referred to as the staircase policy is proposed. The performance of this policy is evaluated by comparing its results with a tight lower-bound and the optimal policy obtained by an MILP model. The results show that the staircase policy can generate near-optimal solutions with an average gap lower than 1.89% for the instances with a planning horizon longer than 30 years. The asymptotic performance of the staircase policy has also been studied. It is proved that the policy is asymptotically near-optimal. Moreover, it is shown that for the special case of $\frac{KN}{D} \in \mathbb{N}$, the staircase policy is optimal when the time horizon is infinite.

Note that this study specifically focuses on problems in which the demand is known in advance and remains constant throughout the planning horizon. The cases involving varying or stochastic demand have been identified as potential future research directions.

References

Ashizawa, R. and J. Lu (2022). Joint maintenance and load-sharing optimization for two-unit systems with workload-dependent deterioration. *Total Quality Science 8*(1), 40–50.

Basciftci, B., S. Ahmed, and N. Gebraeel (2020). Data-driven maintenance and operations scheduling in power systems under decision-dependent uncertainty. *IISE Transactions 52*(6), 589–602.

Dilaver, H. M., A. Akçay, and G.-J. van Houtum (2023). Integrated planning of asset-use and dry-docking for a fleet of maritime assets. *International Journal of Production Economics 256*, 108720.

Drent, C., M. Drent, J. Arts, and S. Kapodistria (2023). Real-time integrated learning and decision making for cumulative shock degradation. *Manufacturing & Service Operations Management 25*(1), 235–253.

Olde Keizer, M. C., R. H. Teunter, J. Veldman, and M. Z. Babai (2018). Condition-based maintenance for systems with economic dependence and load sharing. *International Journal of Production Economics 195*, 319–327.

Uit Het Broek, M. A., R. H. Teunter, B. De Jonge, and J. Veldman (2021). Joint condition-based maintenance and load-sharing optimization for two-unit systems with economic dependency. *European Journal of Operational Research 295*(3), 1119–1131.

Uit Het Broek, M. A., R. H. Teunter, B. de Jonge, J. Veldman, and N. D. Van Foreest (2020). Condition-based production planning: Adjusting production rates to balance output and failure risk. *Manufacturing & Service Operations Management 22*(4), 792–811.

Acknowledgement

This publication is part of the project “Maritime remote control tower for service logistics innovation (MARCONI)” (project number 439.18.309) of the research program “Integrator - Logistics as enabler for enhancing society”, which is (partly) financed by the Dutch Research Council (NWO).